Assignment VIII, PHYS 230 (Modern Physics)
Fall 2019 Due Thursday November 14th, 2019 at Start of Class

As always, turn your legible and complete answers in on separate paper. Remember, I can't give partial credit unless I can follow what you've done. Including words is usually a good thing for you.

The last problem involves you writing three different MATLAB functions; as usual, please email them to me at LarsenML@gmail.com

Since we are dealing with some abstract stuff, the homework this week is a bit more straightforward than normal – mostly problems out of the text.

1. (Problem 6-3 from your text) In a region of space, a particle has a wave function given by $\psi(x) = A\exp(-x^2/2L^2)$ and energy $\hbar^2/2mL^2$, where L is some length. (a) Find the potential energy as a function of x, and sketch (i.e. graph) V versus x. (b) What is the classical potential that has this dependence?

2. (Problem 6-10 from your text) A particle is in the ground state of an infinite square well potential given by Equation 6-21:

$$V(x) = \begin{cases} 0 & 0 < x < L \\ \infty & x < 0 \text{ and } x > L \end{cases}$$

   Find the probability of finding the particle in the interval $\Delta x = 0.002L$ at (a) $x = L/2$, (b) $x = 2L/3$, and (c) $x = L$. (Since Δx is very small, you need not do any integration – the probability is effectively equal to $\psi^*(x)\psi(x)\Delta x$).

3. (Problem 6-11 from your text) Do problem 6-10 for a particle in the second excited state ($n = 3$) of an infinite square well potential.

4. Use your answers from the above two problems to discuss the large n limit (i.e. what happens when the correspondence principle applies). What would you expect your answers to parts (a), (b), and (c) from the above two problems to be when $n \to \infty$?

5. (Problem 6-18 from your text) Suppose a macroscopic bead with a mass of 2.0 grams is constrained to move on a straight frictionless wire between two heavy stops clamped firmly to the wire 10 cm apart. If the bead is moving at a speed of 20 nanometers per year (i.e. to all appearances it is at rest), what is the value of its quantum number, n?

6. (Problem 6-56 from your text) For the wave functions:

$$\psi(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right) \qquad n = 1, 2, 3, \ldots$$

corresponding to an infinite square well of width L, show that:

$$\langle x^2 \rangle = \frac{L^2}{3} - \frac{L^2}{2n^2\pi^2}$$

MATLAB problem on following page!

7. The normalized eigenfunction solutions to the infinite square well potential are as follows:

$$\psi_n(x) = \begin{cases} \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right) & 0 \leq x \leq L \\ 0 & x < 0 \text{ or } x > L \end{cases}$$

with the energies associated with each state equal to:

$$E_n = \frac{n^2\hbar^2\pi^2}{2mL^2}$$

a) Build a MATLAB function named `YourLastName_boxenergy.m` that takes in m, L, and n as inputs and outputs the energy associated with the level *in Electron Volts!*. In other words, after I have saved your file, I should be able to type `YourLastName_boxenergy(2.3e-15,1e-8,37)` and your code will return the energy in electron volts of a particle with a mass of 2.3×10^{-15} kg in a one dimensional infinite potential well with width 1×10^{-8} m and in the 37th energy state. (It should also work for other values of m, L, and n).

b) Use MATLAB to write a different function named `YourLastName_boxwidth.m` that takes in an energy (in eV) and outputs the box width for an electron to have the specified energy as the ground state energy. For example, if I were to type `YourLastName_boxwidth(13.6)` it should give back 1.66×10^{-10}. (This time you don't have to explicitly write units in your responding result; I will assume the answer is in meters). Your code should also work for other energies.

c) Use MATLAB to generate a function named `YourLastName_infwell.m` that takes in three arguments – n, L, and a. n indicates the state of the wave-function (e.g. $n = 1$ indicates the ground state, $n = 2$ indicates the next state, etc.). L indicates the width of the one-dimensional infinite square well (in meters). a is a variable that can take the value 0 or 1; it is equal to 0 if you want to plot the wave function itself, and a is equal to 1 if you want to plot $\psi^*(x)\psi(x)$. In either case, you are plotting the wave function (or the probability) as a function of L for a particle in state n. For example, you would be able to use the code you write to generate any of the different parts of Figure 6-4 in your text. Running your code by typing `YourLastName_infwell(2,1,1)` would produce a plot like the one on the right side of the second row of figure 6-4 (except L would be replaced with 1 everywhere).