Assignment VIII, PHYS 301 (Classical Mechanics)
Spring 2017
Due 4/7/17 at start of class

1. The orbit of a particle moving in a central field is a circle passing through the origin and obeying the equation $r = r_\circ \cos\theta$. The underlying force law is of the form $F(r) \propto r^n$. Find n. (Hint: It is an integer. Second hint: it is not -3).

2. A particle moves in a spiral orbit given by $r = a\theta$ (with a a constant). If $\theta = kt^1$ (with k constant), is the force a central force? If so – how do you know? If not, then $\theta = kt^\xi$ (with k constant and $\xi \neq 1$) DOES correspond to a central force for some value of ξ (still with the spiral orbit described by $r = a\theta$). Find the value of ξ.

3. A point mass moves in a potential of the form $U(r) = \frac{-C}{3r^3}$ with C some constant.

   a) Given ℓ, find the maximum value of $U_{\text{eff}}(r)$. Your answer should be in terms of ℓ, C, and m only.

   b) Let a particle come in from infinity with initial speed $v_\circ$ and impact parameter b. In terms of C, m, and $v_\circ$, there exists some maximal value of b (call it b_{max}) for which the particle is just barely captured by the potential. Your goal is to find the "cross-section" σ for capture in this potential. $\sigma = \pi b_{\text{max}}^2$, so you will need to first find b_{max}. [Recall, the "impact parameter" b of a trajectory is defined to be the closest distance to the origin the particle would achieve if it moved in the straight line determined by its initial velocity far from the origin.]

4. A comet moves in a parabolic orbit lying in the plane of the Earth's orbit. Regarding the Earth's orbit as circular and of radius a, show that the points where the comet intersects the Earth's orbit are given by:

$$\theta = \pm \cos^{-1}\left(\frac{2p}{a} - 1\right)$$

   where p is the perihelion distance of the comet defined at $\theta = 0$.