**Homework 8, PHYS 415 (Fluid Mechanics)**
**Spring 2019**
**Due Thursday 14 March 2019 at Beginning of Class**

As always, turn in your legible and annotated work on separate paper. Note the due date!

1. According to the first garden hose I googled, a typical garden hose has a 5/8 inch diameter and runs 50 feet long. For this problem, assume the flow of liquid through the tube remains laminar (even though in practice the scenario I described will not be laminar and what we've learned won't actually work like you think).

   a) What is the hydraulic resistance of this hose? Assume the fluid is liquid water at 25°C. (Not 20°C!)

   b) What is the absolute pressure necessary on the spigot end of the hose if you want to move 10 cubic feet per minute through this hose (oriented perfectly horizontally)? Assume the hose emits into air in Denver, Colorado (elevation 1609.3 m)) and use the scale height of the atmosphere as 8100 meters.

   c) What is the maximum water flow velocity within the tube for part (b) above? (Again, we're assuming laminar flow, even though the truth is this flow would be turbulent).

2. In class, we either have stated or will state that the solution to Stokes' First Problem of a flat plane at $y = 0$ abruptly starting to move with velocity $U\hat{x}$ at time $t = 0$ beneath a semi-infinite fluid volume with viscosity $\nu$ has solution:

$$v_x(y) = U \left[1 - \text{erf}\left(\frac{y}{2\sqrt{\nu t}}\right)\right],$$

   where erf($x$) is defined by $\frac{2}{\sqrt{\pi}} \int_0^x e^{-\lambda^2} d\lambda$. In class we focused more on the limits of this expression, but this problem will have us numerically explore this solution through computations and plots. Throughout, assume $U = 10$ m/s and $\nu = 1 \times 10^{-6}$ m$^2$/s (matching water at about 20°C).

   a) On the same axis (but with different colors or other easily identifiable features to distinguish the different times) use computer software to plot $v_x(y)$ as a function of $y$ between 0 and 100 meters for $t = 1$ s, $t = 1000$ s, and $t = 10^6$ s. Make sure the $x$-axis of your figure (the $y$ coordinate) is logarithmic so that you can actually see the features in the plots. Make sure to label your axes clearly! We're looking for high quality figures here!

   b) Determine how long you would have to wait so that $v_x(y = 10 \text{ m}) = 0.9U$.

   c) Repeat part (b), except assume that the fluid in question is honey ($\nu \approx 2.2 \times 10^{-3}$ m$^2$/s).

3. Recently we explored some particular famous one-dimensional flows in class. This problem allows us to investigate one of these systems in more detail, both to ensure we understand what's going on and to develop a sense of scale. Developing a sense of scale is important in fluids, since we end up rescaling problems frequently. Thus, even though I don't like having solutions with numerical values any more than you like trying to find a calculator or computer program to plug things in, there's a reason we're doing this stuff.
We start with a Poiseuille-Couette flow, where – like in class – two parallel walls are kept a distance $a$ apart. We assume all pressure gradients and flows are in the $x$ direction. The fluid is incompressible and has viscosity $\eta$. You may assume $\frac{\partial P}{\partial x}$ is constant and equal to $-\kappa$. (Be careful about signs, though! We're going to talk about a pressure gradient both ways in this problem. When $\kappa$ is positive, this means that there are higher pressures on the left-hand side of the system). The bottom wall is stationary, and the upper wall moves with constant velocity $V_\circ$ to the right.

   a) Find the $y$ coordinate where the $x$ velocity is maximum.
   b) Use your answer to part (a) above to determine the maximum velocity of the flow. Make sure to fully simplify your answer!
   c) Derive an expression for the flow rate per unit width in this channel. (Note – make sure your answer has the correct units!) Fully simplify your answer!
   d) Numerically evaluate your answer to part (b) above to find the max speed of a fluid where a total pressure difference of 3 millibars is applied over a horizontal distance of 1 km. The fluid in the layer is water at 25° (not 20°!) Celcius, and the top plate moves with respect to the bottom plate at 10 cm/second; the top plate is 4 cm from the bottom plate. For this part of the problem, assume that the pressure is higher on the left hand side than the right hand side, so that both the pressure gradient force and the top layer of the container both are giving the fluid a force in the +x direction. (Keep 4 significant figures in your answer).
   e) Use a computer program to draw the $x$ velocity as a function of $y$ for the scenario described in part (d) above. Make sure all your axes have labels. You may want to verify your answer to part (d) is consistent with your plot.
   f) Numerically compute the total flow rate per unit width in the scenario outlined in part (d) above.
   g) Find the max velocity of the fluid anywhere in the layer similar to what you did in part (d) above, but this time assume that the pressure gradient is aligned so that the pressure gradient force pushes the fluid to the left while the top layer still pulls the fluid to the right. (All quantities are the same, but we're flipping the sign of $\kappa$).
   h) Repeat your plot done in part (e) above, except for the scenario outlined in part (g).
   i) Now align your pressure gradient force as constructed in part (g), so that the pressure gradient opposes the Couette component of the flow. Find the total horizontal pressure gradient required so that the *net flow* would be 0. (There will still be flow to the right near the top plate, but this will be cancelled by flow to the left near the bottom plate). Note – I'm looking for an actual number here – not a symbolic answer! Leave the rest of the parameters of the problem the same as they were defined above.

4. Consider the coaxial cylinders shown below. The inner (solid) cylinder has diameter $D_1$ and the outer (hollow) cylinder has diameter $D_2$, and we choose the $z$ axis as the central axis of both cylinders, pointing to the right. Both cylinders are stationary (the walls don't move). In the annular region between these cylinders is an incompressible fluid with viscosity $\mu$, and the fluid is induced to move in the positive $z$ direction in the annulus due to the presence of a constant horizontal pressure gradient $-\kappa$.

   a) Find an expression for the steady $z$ velocity in the annulus as a function of $s$ (the distance from the axis of both cylinders); your answer should depend on $\mu$, $\kappa$, $s$, $D_1$, and $D_2$ only. You may assume $v_s = v_\phi = 0$, and that $v_z$ is a function of $s$ only. Also, you may neglect gravity or any other body forces in the $z$ direction ($f_z = 0$). Your answer will be a little bit messy; please simplify it as much as possible. (That will likely help in the rest of the problem).

   b) Find the $s$ coordinate of the maximal velocity.

   c) Use a computer program to draw the $z$ velocity as a function of $s$ in the annulus. Let $\kappa$=0.01 Pa/m, $\mu = 1 \times 10^{-3}$ kg/(m s), $D_1 = 0.1$ m, and $D_2 = 2$ m. Make sure all your axes have labels.

   d) Use your plot to determine the maximum velocity of flow in the scenario outlined in part (c) to at least 3 significant figures. (If you are a masochist, you could try to do it analytically – but when I tried to do that I got lost in a sea of logarithms).