Assignment IX, PHYS 230 (Introduction to Modern Physics)
Fall 2015
Due Thursday, 11/12/15 at start of class

1. A particle at some time has the following wave equation:

$$\psi(x) = \begin{cases} 0 & x < 0 \\ Ae^{-kx} & x \geq 0 \end{cases}$$

with k and A unspecified (real and nonzero) constants.

   a) This $\psi(x)$ is not a valid solution for any real potential-energy function. How can you tell?

   b) If we ignore the issue pointed out in part (a), we can still treat this as a normal wave-function. What must A be if the wave-function is properly normalized?

   c) Given your answer to part (b), what is the probability that the particle is between $x = \frac{1}{k}$ and $x = \frac{3}{k}$?

2. In a region of space, a particle has a wave function given by $\psi(x) = Ae^{\frac{-x^2}{2L^2}}$ and has energy $E = \frac{\hbar^2}{2mL^2}$ where L is some length. Find the potential energy as a function of x and sketch (i.e. graph by hand) $U(x)$ vs. x.

3. Suppose a bead with mass of 3.5 grams is constrained to move on a straight, frictionless wire between two heavy stops clamped firmly to the wire 12 cm apart. If the bead is moving at a speed of 15 nanometers per year (i.e. to all appearances it is at rest), what is the value of its quantum number n?

4. For the wave functions:

$$\psi(x) = \begin{cases} \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right) & 0 < x < L \qquad n = 1, 2, 3, \ldots \\ 0 & \text{otherwise} \end{cases}$$

corresponding to an infinite square well of width L:

   a) Find $\langle x \rangle$ (the mean expected position). (Your answer may depend on n).

   b) Find $\langle x^2 \rangle$ (Your answer may depend on n).

5. Classically, if we just threw a particle in a box and knew nothing else about it, we might hypothesize that its wave function would look something like the following:

$$\psi_1(x) = \begin{cases} A & 0 < x < L \\ 0 & \text{otherwise} \end{cases}$$

(Technically, this isn't a valid wave-function since it isn't continuous. However, the idea that "the particle is equally probably anywhere in the box" is how we would normally interpret this physically. If the fact that this wave-function isn't allowed *really* bothers you, we can write this as:)

$$\psi_2(x) = \lim_{\alpha \to 0} \begin{cases} 0 & x < 0 \\ \frac{Ax}{\alpha} & 0 < x < \alpha \\ A & \alpha < x < L - \alpha \\ \frac{A}{\alpha}(L - x) & L - \alpha < x < L \\ 0 & x > L \end{cases}$$

a) If we assume that $\psi_1(x)$ is a valid wave-function for this scenario, find A so that this is a properly normalized wave-function.

b) Find $\langle x \rangle$

c) Find $\langle x^2 \rangle$ and compare to what your answer in part (b) of the previous problem would be as $n \to \infty$ (they should match).

6. Let us define the following wave function:

$$\psi(x) = \begin{cases} 0 & x < -L \\ (C - Ax) & -L < x < 0 \\ D & 0 < x < L \\ G + Hx & L < x < 2L \\ 0 & x > 2L \end{cases}$$

with A, C, D, G and H unknown constants and L some specified length.

a) Use continuity of ψ at boundaries to write the above wave function in terms of just D, L, and x.

b) Use the normalization condition ($\int_{-\infty}^{\infty} |\psi(x)|^2 \mathrm{d}x = 1$) to determine what D must be in terms of L.

c) Use the above answers to determine the probability of the particle being between $\frac{L}{2}$ and L.