Assignment IX, PHYS 230 (Introduction to Modern Physics)
Spring 2017
Due Wednesday, 4/6/17 at start of class

Recall that the definition of "phase velocity" $v_p = \frac{\omega}{k}$ and that the definition of "group velocity" is $\frac{\mathrm{d}\omega}{\mathrm{d}k}$.

1. Show that $v_g = v_p + k\frac{\mathrm{d}v_p}{\mathrm{d}k}$

2. In order to locate a particle to within 5×10^{-12} meters using light, the wavelength of the light must be at most 5×10^{-12} meters.

   a) Calculate the energy of a photon with $\lambda = 5 \times 10^{-12}$ m.

   b) Calculate the momentum of a photon with $\lambda = 5 \times 10^{-12}$ m.

   c) If this light bounces off an electron leaving an uncertainty $\Delta x = 5 \times 10^{-12}$ m to its position, what is the minimum uncertainty in the electron's momentum?

3. An excited state of a certain nucleus has a half-life of 2.3 ns.

   a) Taking this to be the uncertainty Δt for emission of a photon, calculate the minimum uncertainty in the frequency of the emitted light.

   b) If the emitted light is expected to have a wavelength 0.05nm, what is $\Delta f/f$ for this light? ($\Delta f/f$ can be interpreted as the fractional uncertainty of the frequency).

4. Wave functions must be "normalized". In other words, the integral:

$$\int_{-\infty}^{\infty} \Psi^* \Psi \mathrm{d}x = 1$$

   for a 1-dimensional system. Recall that Ψ^* indicates the complex conjugate and, if Ψ doesn't have any complex quantities (no imaginary numbers), then $\Psi^* = \Psi$. Let $\Psi(x, 0) = A|x|e^{(-k|x|)}$, with A and k unspecified constants. Find what A has to be in terms of k so that the wave function is properly "normalized". (You may use on-line resources, integral tables, and/or computational tools to do the necessary integral for you. Be careful with the absolute values, though).

5. If we know that the integral $\int_{-\infty}^{\infty} \Psi^* \Psi \mathrm{d}x = 1$, what do the units of Ψ have to be?

6. The wave function describing a state of an electron confined to move along the x axis is given at time zero by:

$$\Psi(x, 0) = Ae^{-x^2/(4\sigma^2)}$$

   a) Where is the electron most likely to be found?

   b) What is the probability of finding the electron in a region $\mathrm{d}x$ centered at $x = 0$?

   c) What is the probability of finding the electron in a region $\mathrm{d}x$ centered at $x = \sigma$?

   d) What is the probability of finding the electron in a region $\mathrm{d}x$ centered at $x = 2\sigma$?