Assignment IX, PHYS 272 (MAP)
Fall 2014
Due 11/14/14 at start of class

1. Solve the following differential equation:

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} + 4x = \sin(2t) \qquad x(t=0) = 7 \qquad \frac{\mathrm{d}x}{\mathrm{d}t}(t=0) = 0$$

2. Solve the following differential equation:

$$\frac{\mathrm{d}^2z}{\mathrm{d}t^2} + \frac{\mathrm{d}z}{\mathrm{d}t} - 5z = \mathrm{e}^{2t} \qquad z(t=0) = 1 \qquad \frac{\mathrm{d}z}{\mathrm{d}t}(t=0) = 2$$

3. A very important partial differential equation in Physics is the wave equation. Here's an example of an equation with the same form as the wave equation:

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{16}\frac{\partial^2 u}{\partial t^2}$$

With $u = u(x,t)$. Let us write $u(x,t) = X(x)T(t)$ and try to solve this differential equation by separation of variables.

a) Use separation of variables to get to the following step:

$$\frac{1}{X}\frac{\mathrm{d}^2X}{\mathrm{d}x^2} = \frac{1}{16T}\frac{\mathrm{d}^2T}{\mathrm{d}t^2} = -\kappa^2$$

with κ an unknown constant.

b) Solve the resulting ordinary differential equation for X in terms of κ and 2 undetermined constants.

c) Solve the resulting ordinary differential equation for T in terms of κ and 2 other undetermined constants.

d) Combine your solutions to parts (b) and (c) [multiply them together] to get a tentative answer for $u(x,t)$. We'll now refine this solution by applying the boundary conditions and initial conditions one at a time.

e) The first boundary condition is that $u(x,t) = 0$ when $x = 0$. Use this initial condition to simplify your answer in part (d).

f) The second boundary condition is that $u(x,t) = 0$ when $x = 2$. Use this initial condition to constrain the value of κ to be equal to $n\pi/2$ with n some integer. Rewrite your answer found in part (e) in terms of n instead of κ.

g) Another boundary/initial condition to this system is that $\frac{\partial u}{\partial t}$ must vanish at $t = 0$. Use this to simplify the answer further. Your answer should now be of the form $u = A\sin(\alpha n x)\cos(\beta n t)$ though you should have other things in there for α and β.

h In reality, the above solution you found in part (g) is a valid solution for any of a number of different values of n. We *should* write the solution as $u = \sum_{n=0}^{\infty} A_n \sin(\alpha n x)\cos(\beta n x)$. If we state that at $t = 0$, the solution is $u(x, t = 0) = 8\sin(\pi x) - 5\sin(3\pi x)$, we have enough information to find all of the A_ns. Use this information to write down the final answer for $u(x,t)$ subject to all of the initial and boundary conditions outlined above.

i) Verify your answer in part (h) satisfies the original differential equation.

> The next area of the course will be probability and statistics. We may not have done much of this yet, but a good chunk of this content is common-sense and careful accounting for possibilities. Be careful with the following; you do know enough to figure this out...but it is a case where you might have to develop a method on your own instead of just applying one that has previously been given to you. Work the following questions carefully. If you need help, talk to me or one of your classmates.

4. Probability is an area where your intuition may often lead you wrong. Read the following questions very carefully. Give your answer and (if you seek partial credit) an explanation. Note, however, that wrong answers will garner at most half credit.

   a) Jane flips 4 (fair) coins. What is the probability of the final outcome of this procedure being three heads and a tail? (On all coin-throwing/flipping questions, assume that there is 0 probability of anything except heads or tails and, if the coins are fair, you may assume the probability of a heads or a tails for any given throw is equiprobable).

   b) Alfred flips 4 (fair) coins and looks at the result. He tells you (honestly) that at least two of the coins came up heads. What is the probability that the outcome of his throw was three heads and a tail?

   c) Nancy has 3 children. Each is either a girl or a boy, and we assume each birth is a 50/50 proposition. She tells you she has at least 2 girls. What is the probability she has a boy and two girls?

   d) What is the probability of being dealt a hand of "blackjack" (two cards total; one card an ace and the other card a ten, jack, queen, or king; it doesn't matter if the ace or the face is first) if....

      i) One deck of cards is used?

      ii) Eight decks of cards are used?

5. Let's invent a game. If you roll two 6-sided dice, there are 11 possible outcomes (2,3,4,5,6,7,8,9,10,11, or 12). 5 of these 11 outcomes are prime numbers. Therefore, let us say you bet $1. If you roll a prime number, I give you $2.20 (11/5 of $1). If you don't roll a prime number, I keep your dollar.

   a) Who has the edge in this game? You or me?

   b) For each $1 you bet, how much do you expect to win (or lose) if we play repeatedly for a long time?

   c) How could you modify the payout scheme so that the game is fair? (No expected gain or loss for either you or me)?