**Assignment IX, PHYS 308**
**Fall 2016**
**Due 11/18/16 at start of class**

NOTE: Please leave your answers in terms of actual numbers (with appropriate units) when possible. Please provide full, legible, easy to follow solutions to the following problems. I can't give you credit if I can't follow your reasoning. Extensive exposition on your thought process or strategy is always appreciated.

1. In class, we typically only drew one Köhler figure at a time. It is pretty common to see multiple traces on the same plot, with different dry radii, chemical compounds, temperatures, etc. on the same plot. [Why does the dry radius matter? That ends up influencing the Raoult effect for all future sizes]. On the top of the following page, I have presented a figure (from Seinfeld and Pandis) which includes curves for three different starting (dry) diameters for two different compounds. (The labels may be hard to read, so the solid line corresponds to NaCl while the dotted lines correspond to $(NH_4)_2SO_4$). Use the plot on the following page to answer the following questions. *In parts (a-f) below, indicate if the particle completely evaporates away, stays stably in solution at a particular diameter (and identify the associated stable diameter), or grows continuously!*

   a) A diameter=0.1 micron dry Ammonium Sulfate aerosol is exposed to an ambient supersaturation of 0.05% at 293K. What happens to it?

   b) A diameter=0.05 micron dry NaCl aerosol somehow activates to form a water droplet that grows to a diameter of 1 micron. This drop is then exposed to a supersaturation of 0.1% at 293K. What happens to it?

   c) A diameter=0.05 micron dry NaCl aerosol somehow activates to form a water droplet that grows to a diameter of 0.3 micron. This drop is then exposed to a supersaturation of 0.0% at 293K. What happens to it?

   d) A diameter=0.1 micron dry Ammonium Sulfate aerosol somehow activates to form a water droplet of diameter 10 microns. This drop is then exposed to a supersaturation of 0.08% at 293K. What happens to it?

   e) A diameter=0.1 micron dry NaCl aerosol somehow activates to form a water droplet that grows to a diameter of 0.5 microns. This drop is then exposed to a supersaturation of 0.15% at 293K. What happens to it?

   f) A diameter=0.05 micron dry NaCl aerosol somehow activates to form a water droplet that grows to a diameter of 0.5 microns. This drop is then exposed to a supersaturation of 0.15% at 293K. What happens to it?

   g) Explain why the critical supersaturation grows with decreasing dry diameter.

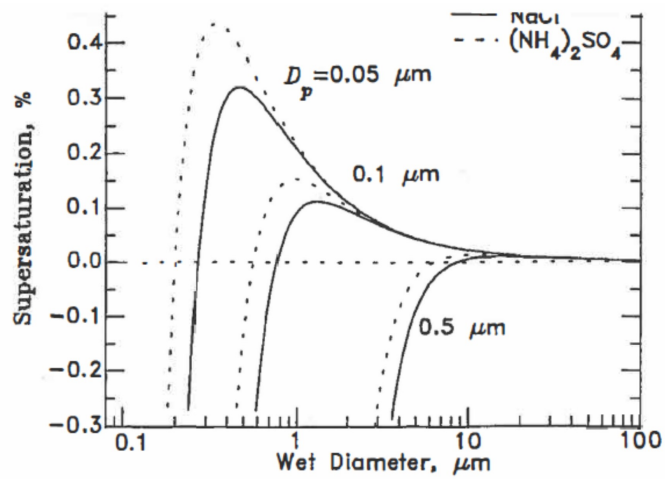


**FIGURE 15.5** Köhler curves for NaCl and $(NH_4)_2SO_4$ particles with dry diameters 0.05, 0.1, and 0.5 $\mu$m at 293 K (assuming spherical dry particles). The supersaturation is defined as the saturation minus one. For example, a supersaturation of 1% corresponds to a relative humidity of 101%.

2. Recall that the equation governing condensational growth can often be written:

$$r\frac{\mathrm{d}r}{\mathrm{d}t} = GS$$

with $G$ hiding a whole bunch of quantities related to the energetics/thermodynamics of the phenomena, the environment of the droplet, and material constants.

Assume you have a freshly minted water droplet that formed on a 0.05 micron (radius) soluble particle. The radius of the water droplet grows to about 1 micron radius in about a second, and the supersaturation is about 0.008. (This is realistic).

a) Based on the information above, what would the value of $G$ be? (You can either use a class-derived equation to do this, or you can multiply both sides the equation above by $\mathrm{d}t$ and integrate to find $r(t)$.)

b) Assuming $G$ retains this value throughout the process, how long would it take this drop to grow via condensation to the size of a typical raindrop (radius around 400 micrometers)?

3. A better (more realistic, but much harder to solve for $r$) equation that describes the rate of droplet growth can be written:

$$r\frac{\mathrm{d}r}{\mathrm{d}t} = G\left(S - \frac{a}{r} + \frac{b}{r^3}\right)$$

does the equation used in the previous problem (a.k.a. $r\frac{\mathrm{d}r}{\mathrm{d}t} = GS$) over- or under-estimate the drop growth rate for a water droplet for $S > S_{\mathrm{crit}}$ and $r \approx r_c$? Justify your answer with a coherent explanation.

4. In class, we have argued (or will soon assert) that the condensation process tends to cause a distribution of droplet sizes to become less dispersed (in other words, condensation narrows the size distribution). This is because small droplets grow faster than large droplets, and ultimately allows the small drops to partially "catch up" to the larger ones. At some point, we will talk about gravitational coagulation, which follows this basic form:

$$\frac{\partial m}{\partial t} \propto \begin{cases} r^6 & \text{for} 10 \leq r \leq 50\mu \text{ m} \\ r^3 & \text{for} r \gtrsim 50 \ \mu\text{m} \end{cases}$$

Based on this, figure out if gravitational coagulation should cause a size distribution to become narrower or wider. Assume the density of water is constant, and the mass of a drop is equal to $\rho_{\text{water}} V$.