Homework 9, PHYS 415 (Fluid Mechanics)
Spring 2019
Due Thursday 28 March 2019 at Beginning of Class

As always, turn in your legible and annotated work on separate paper. Note the due date!

1. An airplane is fitted with a Pitot tube to help determine its true air speed. For this problem, assume that surface air pressure at sea level everywhere is 101325 Pa and that air density at the surface is 1.24 kg/m^3. Assume air pressure and density decay exponentially with height with a scale-height of 8100 meters.
   a) Assume a plane flies at a speed relative to air of 500 miles per hour at an altitude of 40000 feet. What are the two pressure readings given by the Pitot tube in Pa?
   b) Derive a formula that takes in the higher of the two pressures from the Pitot tube (call it P_1) and reports back the altitude (in meters) of the plane.
   c) Derive another formula that takes in both pressures P_1 and P_2 (with $P_1 > P_2$ as in part (b) above) and determines the true air-speed of the plane. Note – your expression is not allowed to have the altitude in it explictly, though your answer to part (b) might help you out here.
   d) Use computer software of some sort to plot v as a function of P_1 and P_2 as a surface plot. Limit yourself to reasonable domains such that the altitude is less than 14000 meters and, of course, $P_1 > P_2$. Neglect any shock-wave like effects that might be present. Give me two versions of the plot; one as a top-down view with colors corresponding to the value of v, and another that's rendered as a 3d surface with the v axis logarithmic.
2. One classic application of Bernoulli's integral that we intentionally didn't talk about in class is the famous problem of an inviscid fluid draining out of a small circular hole in a reservoir. You may want to read your textbook's treatment on pages 154-156 to help attack this problem.
   Assume you have a bucket that is a right-circular cylinder of radius R and height H filled to its very top with an incompressible ideal fluid of density $\rho_\circ$ near the surface of the Earth. At time $t = 0$, a small hole of radius r is opened along the side wall of the bucket, some distance $h < H$ above the bottom of the bucket. (Assume the opening actually takes the shape of Borda's mouthpiece as described in your text). Show that $v_f(t) = \sqrt{2g(H-h)} - \frac{r^2tg}{2R^2}$ where $v_f(t)$ is the velocity of the minimal cross-section of the expunged jet of flouid, as described in the text.

   (MORE ON BACK)

3. Note: You are welcome to solve (all parts of) the following problem numerically (a.k.a. computationally). There should be a closed-form analytical solution, but my guess is that the integrals will get gnarly in parts (a) and (b); parts (c) and (d) are straightforward to do analytically (look at your answer to problem 2 above for help), but since I already had a code written, I solved parts (c) and (d) numerically as well. That being said, if you have no idea how to do this with computer code (or want to challenge yourself with integration), by all means attack part or all of this problem without a computational aid. For what it is worth, I am writing my own solution in MATLAB.

   This is somewhat similar to the last problem, but the geometry is a bit different. My grandfather (Ernest Larsen, if you want to put a name to your frustrations in this problem) used to store gasoline in a suspended cylindrical 55-gallon drum (turned on its side) elevated about 8 feet above the ground. According to Dr. Google, the inner dimensions of these containers are about 572 mm diameter and 851 mm length. Assume that this drum is completely filled with unleaded gasoline (density of 708 kg/m^3 – though that won't matter – and assumed inviscid and incompressible for this problem). Assume also there's a super-tiny air-leak in the top of the container so that no vacuum is created within the container as it drains. Assume the air outside the tank is massless. Finally, assume the circular opening that can be opened in this container has Borda's mouthpiece shape. Hint: the area of a circle of radius R when filled from its bottom to some height $h \leq 2R$ can be computed from:

$$A = R^2 \cos^{-1}\left(\frac{R-h}{R}\right) - (R-h)\sqrt{2Rh - h^2}$$

   where the arc-cosine is assumed to return a value in radians.

   a) If the opening is 2 inches in diameter and situated on the curved side of the drum pointing directly downward, how long would it take to drain the drum?
   b) If the opening is 4 inches in diameter and in the same geometry as outlined above, how long would it take to drain the drum?
   c) Let's say that the drum was mounted upright instead of on its side. If the drain plug is again in the bottom and the opening is 2 inches in diameter, how long does it take to drain the drum now?
   d) Again, let's say the drum is mounted upright and the opening is on the bottom of the drum and 4 inches in diameter , how long would it take to drain the drum?