Assignment X, HONS 157 (Honors Physics I)
Fall 2015
Due Friday, 11/13/15 at start of class

1. Two astronauts, each with mass M, are connected by a rope of length d having negligible mass. They are isolated in space, orbiting their center of mass with each moving at speed v. Calculate:

   a) The magnitude of the angular momentum of the system (in terms of M, v, and d). (Assume the astronauts are point particles).

   b) The rotational energy of the system (in terms of M, v, and d).

   By pulling the rope, the astronauts shorten the distance between them to $d/2$.

   c) What is the new angular momentum of the system? (again, in terms of M, v, and d).

   d) What are their new speeds?

   e) What is the new rotational energy of the system?

   f) How much work is done by the astronauts in shortening the rope?

2. At times, massive astronomical bodies can suddenly collapse. Let's let a spinning star have initial moment of inertia $I_\circ$. If, suddenly, the star collapses in such a way that now the star has rotational inertia $\frac{I_\circ}{4}$, what is the ratio of the final rotational kinetic energy to the initial kinetic energy. (Note: $\omega_f \neq \omega_i$).

3. A wedge of height H and wedge-angle $\theta_\circ$ has a number of different items move down it.

   a) If a solid sphere of radius R and mass M starts from rest at the top of the wedge and rolls down the wedge without slipping, how long does it take the sphere to reach the bottom of the wedge?

   b) If a disk of radius R and mass M starts from rest at the top of the wedge and rolls down the wedge without slipping, how long does it take the disk to reach the bottom of the wedge?

   c) If a hoop of radius R and mass M starts from rest at the top of the wedge and rolls down the wedge without slipping, how long does it take the hoop to reach the bottom of the wedge?

   d) If a mass (irrelevant shape) of radius R and mass M starts from rest at the top of the wedge and slides (frictionlessly) down the wedge, how long does it take the mass to reach the bottom of the wedge?

4. Technically, due to the fact that both the Sun and the Earth are pulling you towards the center of the Earth at night and during the day the Sun helps to pull you off of the Earth, you should weigh more at night than during the day. (It turns out, there are other effects that we usually ignore that are way bigger than this....but let's just play with this for a minute or two to see the size of the effect). Let's say – for sake of simplicity – that Earth is not tilted and Chuck Finley stands at the equator. For the sake of this problem, assume Chuck Finley has a mass of 100 kg.

   a) How much does Chuck Finley weigh at noon (assuming that only the gravitational forces from the Earth and the Sun are relevant).

   b) How much does Chuck Finley weigh at midnight (assuming that only the gravitational forces from the Earth and Sun are relevant). (Make sure you've kept enough digits to your answers to parts (a) and (b) so that you have a difference.)

5. Two astronauts each of mass M are in space a distance D apart.

   a) How large is the gravitational force of attraction between the two astronauts?

   b) Using realistic values of $M = 80$ kg and $D = 10$ m, numerically evaluate your answer to part (a) above.

   c) Assuming a constant net force of the magnitude found in part (b) above, how long would it take for an Astronaut to move 5.0 meters? [Note – this is not how long it would take the two astronauts to come together in space due to gravitational attraction, since the gravitational force gets stronger as the two astronauts get closer together. However, this does give you a sense of the necessary time-scales for something like this to happen.]

6. The Earth has a mass of about 5.97×10^{24} kg. Our moon has a mass of about 7.34×10^{22} kg. The average Earth-moon distance varies a bit during the lunar orbit, but averages about 3.8×10^{8} m.

   a) Where is the center of mass of the Earth-moon system? (Make sure your answer is unambiguous).

   b) There is a point somewhere between the Earth and the moon where the gravitational pull from the Earth is exactly balanced by the gravitational pull from the moon in the opposite direction. How far from the center of the moon is this point?

7. Planet X orbits its star Y in an elliptical orbit. You may assume the mass of star Y is much grater than the mass of planet X. The radius of planet X is R_X and the radius of star Y is R_Y. Both are assumed to be spheres of constant density, with the density of the planet being ρ_X and the density of the star being ρ_Y (where ρ_X might not equal ρ_Y). The distance of closest approach between the planet and the star is D_{min} and the furthest distance between the planet and star anywhere in the orbit is D_{max}. The orbital speed of the planet depends on where the planet is in the orbit, but has a maximal value of $v_\circ$. For this problem, leave your answers in terms of the variables above and fundamental constants *only*.

   a) What is the orbital speed of the planet when it is at D_{min}?

   b) What is the orbital speed of the planet when it is at D_{max}?

   c) Let a mass m be placed between the planet and the star when their separation distance is D_{min}. The mass m can be found a distance $5R_X$ from the *surface* of planet X. If, when at that height, the gravitational attraction on the mass from the star is equal in magnitude and opposite in direction to the gravitational attraction to the planet, what must the ratio ρ_x/ρ_y be? (You may assume that $5R_x \ll D_{\text{min}}$).

8. The Earth is on average 1 A.U. (astronomical unit) away from the sun. The Earth goes around the sun once per year. Using just this information and Kepler's third law, use the following mean planet-sun distances to infer the orbital period for each listed planet:

   a) Mercury ($D \sim 0.39$ A.U.)

   b) Venus ($D \sim 0.72$ A.U.)

   c) Mars ($D \sim 1.52$ A.U.)

   d) Jupiter ($D \sim 5.20$ A.U.)

   e) Neptune ($D \sim 30.07$ A.U.)

   f) Pluto ($D \sim 39.49$ A.U. for the "dwarf planet")