**Assignment X, PHYS 111 (General Physics I)**
**Fall 2016**
**Due 11/18/16 at start of class**

As always, please put your clearly written answers on separate paper.

1. An undamped, undriven oscillator reaches maximal displacement from equilibrium $A$ and has maximal velocity $v_\circ$. In terms of these two variables and fundamental constants *only*, what is:

   a) The period of oscillation?

   b) The frequency of oscillation?

   c) The angular frequency of oscillation?

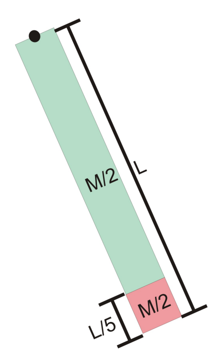
2. Earlier this semester, you conducted a "ballistic pendulum" lab. This problem explores a similar physical setup.

   We will use the figure below to represent a physical pendulum that can be viewed as a simplified version of the ballistic pendulum setup. The physical pendulum will be made of two equal-mass components, each having mass $M/2$. The black circle at the top of the figure indicates the pivot point. The center of mass of the system is designed to be at the intersections of the two pieces of the physical pendulum, and exists at a distance of $4L/5$ from the pivot.

   a) What is the angular frequency ($\omega$) of small oscillations for this system? (You may assume that the pendulum is a very thin rod, so you don't have to worry about the "width" of the arm, just its length). Leave your answer in terms of $L$, $M$, $g$, and any other necessary natural constants.

   b) If we had a simpler system – a simple pendulum with a bob of mass $M$ and an arm of length $4L/5$ – what would be the angular frequency of small oscillations for this system? (Leave your answer again in terms of $L$, $M$, $g$, and any other necessary natural constants).

   c) Which of your answers (a) or (b) is greater?



3. A mass-spring system of mass $M$ and spring constant $k$ is oscillating on a frictionless horizontal table-top at its natural (angular) frequency $\omega_\circ$ with amplitude $A$. At the instant the mass is stretched its furthest from equilibrium, you clip on a second mass $M$ onto the top of the first mass, so now a mass of $2M$ is being oscillated by a spring of spring constant $k$.

   a) What is the ratio $\frac{E(\text{before new mass added})}{E(\text{after new mass added})}$?

   b) What is the ratio $\frac{v_{\max}(\text{before new mass added})}{v_{\max}(\text{after new mass added})}$? ($v_{\max}$ indicates maximal velocity).

   c) What is the ratio $\frac{\text{Oscillation period before new mass added}}{\text{Oscillation period after new mass added}}$?

4. A circular hoop hangs from a nail on a barn wall. The mass of the hoop is 3 kg and its radius is 20 cm. If the hoop is displaced slightly by a passing breeze, what is the period of the resulting oscillations?

5. A classic example of an oscillatory system is someone on a swing on a swing set. We will explore this as a damped (but not driven) simple harmonic oscillator. If you go back and solve the appropriate differential equation for this system, you obtain (assuming it starts at maximum displacement)

$$\theta(t) = \theta_m e^{\frac{-bt}{2L}} \cos(\omega' t)$$
$$\omega' = \sqrt{\frac{4gL - b^2}{4L^2}}$$

   where $\theta_m$ is the initial angle with respect to the equilibrium angle, $L$ is the length between the pivot point and the swinging mass, and $b$ is a constant that characterizes the "damping" of the swing-set.

   a) Briefly explain how you could estimate the damping coefficient $b$ of a swing-set. Explain – in detail – how you could find $b$ if you made some measurements of the system. (You are only allowed to measure lengths, masses, and times.)

   b) Use realistic values for lengths, masses, and times to estimate $b$ for a realistic swing-set.

   c) What would the ratio $\frac{\omega'}{\omega_\circ}$ be for the values you assumed in part (b)? (You may assume that the system both with and without damping is well-approximated as a simple harmonic oscillator. You should not need to calculate any moments of inertia).

   d) (Extra credit). Actually do it. Go find a swing. (There's a few within a couple blocks of our class on Concord street, south of Calhoun – though any real swing will work). Measure the values. Calculate $b$. Find $\omega'$ and compare it to $\omega_\circ$ for the system. (Again, you may assume that the system without damping is well approximated by a simple harmonic oscillator – no need to treat it as a physical pendulum). Feel free to include photographic or video evidence to help justify that you actually did this (may be helpful if you do the extra credit but aren't too confident in your calculations).