Assignment X, PHYS 111 (General Physics I)
Fall 2018
Due 11/16/18 at start of class

As always, please put your clearly written answers on separate paper.

1. I once visited a data-center that used a giant flywheel to store kinetic energy that would enable the entire facility to keep running for a few seconds after a power-outage. The idea behind this is that you use a motor to get a system with a large amount of rotational inertia rotating while the power is still on and keep the thing rotating at all times while the power is on. If the power goes off, the flywheel will still keep turning for a little while and the motion of that flywheel can power your building's electricity through its motion (much like a turbine or water wheel). The details are unimportant for this problem, other than the fact taht you can take the rotational energy of the flywheel and convert it back into electrical energy to power the thing you care about for a little while.

   a) If the flywheel is a horizontally-mounted uniform circular cylinder of density ρ, radius R, and height H, and rotates at a frequency of f rotations per second, how much kinetic energy is stored in the motion of the flywheel? (Leave your answer in terms of variables in the problem statement only).

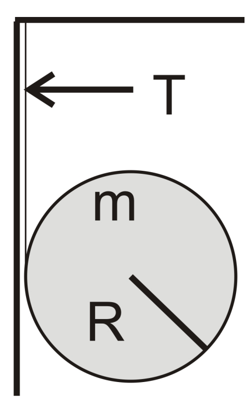
   b) Let's say that the building had to power 300 computers, each requiring about 500 Watts to keep running. You want your flywheel to power everything for 10 seconds – enough time for backup generators to kick on and start running to take over. The flywheel is constructed from a solid steel disk. (The density of steel is 7600 kg/m^3). If the flywheel is 0.1 meters thick, and is rotated at a constant velocity of 3 rotations per second about its center, what would its radius have to be in order to meet the energy requirements to keep the computers running for 10 seconds? Assume all energy stored in the flywheel's motion can be completely converted back to electrical energy with perfect efficiency. (If case you are curious, the data-center's old solution to this problem was to use a bank of several hundred car batteries to do this task for them. That solution ended up being impractical).

   c) What is the magnitude of the angular momentum of the disk in part (b) when it is rotating at 3 rotations per second?

2. A wheel of radius R and mass M can take on a variety of shapes – it can essentially be a hoop (if only very small massed spokes are used to connect the outer edge of the wheel to the axle), it can be very much like a cylinder (or its two dimensional version, a disk) (if there is a continuous mass distribution between the axle and the edge of the wheel), and it can be spherical (with the axle going through the wheel's center). Essentially anything with a circular shadow when illuminated from above can work. That being said, I'm going to claim that the moment of inertia of ANY wheel of mass M and radius R must be less than or equal to MR^2. Give a compelling argument either for or against my claim.

3. A yo-yo can be treated approximately like a uniform disk of mass m and radius R. If the yo-yo string is attached to the ceiling and the yo-yo is allowed to drop under the influence of gravity, find the following: (Note – the picture puts a frictionless wall next to the yo-yo to try and clearly show that the yo-yo falls with the string coming off the side of the yo-yo. There is no interaction with the wall).

   a) What is the tension in the string as the yo-yo falls? (Hint – it will be constant). (Leave your answer in terms of m, R, and/or fundamental constants).
   b) What is the magnitude of the acceleration of the center of the yo-yo? (Leave your answer in terms of m, R, and/or fundamental constants).
   c) What is the speed of the center of the yo-yo after the center has fallen distance h after being dropped from rest? (Leave your answer in terms of h, m, R, and/or fundamental constants).
   d) What is the total translational kinetic energy $\frac{1}{2}mv_{\text{cm}}^2$ after the yo-yo center has fallen distance h? (In terms of m, h, R, and/or fundamental constants).
   e) What is the total rotational kinetic energy $\frac{1}{2}I\omega^2$ after the yo-yo center has fallen distance h? (In terms of m, h, R, and/or fundamental constants).
   f) What is the angular momentum of the yo-yo about its center after the yo-yo center has fallen distance h? (In terms of m, h, R, and/or fundamental constants).



4. A negligibly thin rod of length $2L$ runs along the x-axis from the point $-L\hat{\imath}$ to the point $L\hat{\imath}$. The ends of this rod mark the midpoints of two solid spheres, each with radius $R < L$ (so that there's a sphere of radius R centered at each of the two end points $-L\hat{\imath}$ and $L\hat{\imath}$). (This system looks like an old-school "barbell"). Let the rod have mass m and each sphere have mass M. By construction, the center of mass is at the origin no matter what m and M are.

   a) Find the moment of inertia of the whole system through the axis that goes through the origin and points in the $\hat{\jmath}$ direction. (Note that if you have a compound object, I for the system is equal to the sum of I for the parts of the system. Also note that you will need to use the parallel axis theorem, since we are *not* rotating the spheres about their centers).

   b) Find the moment of inertia of the whole system through and axis pointing in the $\hat{\imath}$ direction.

   c) Introductory texts give the following simple approximation of the nuclear radius: $R \approx (1.2 \times 10^{-15}\text{ m})A^{1/3}$ where A is the number of protons and neutrons in the atom. The bond-length of an N_2 molecule is approximately 145×10^{-12} m. Assuming that the bond itself has negligible mass, calculate the ratio between I_y/I_x for an N_2 molecule using the basic picture described above.