Assignment X, PHYS 230 (Introduction to Modern Physics)
Fall 2015
Due Tuesday, 11/24/15 at start of class

1. Read Chapter 6 of your text. (Seriously. Read it. No joke.)

2. A particle at some time has the following wave equation:

$$\psi(x) = \begin{cases} Axe^x & 0 \leq x \leq 1 \\ \frac{B}{x^3} + Cx^4 & x > 1 \end{cases}$$

   A, B, and C unspecified (real) constants. Note that this is only defined for positive x. We are assuming that $x < 0$ is, for some reason, unavailable in this problem. You do not have to worry about boundary conditions at $x = 0$ or $x \to -\infty$. We know the particle is between $x = 0$ and $x = +\infty$.
   Rewrite the wave function as a properly normalized wave-function without any undetermined constants. (Ignore the criterion that the derivative of the wave-function must be continuous at $x = 1$).

3. Let a particle of mass m be bound inside the potential well defined via:

$$U(x) = \begin{cases} \infty & x < 0 \\ 0 & 0 \leq x \leq L \\ U_\circ & x > L \end{cases}$$

   Since this is a bound state, you may assume $E < U_\circ$.

   a) Follow the same basic methods I used in class to write down solutions for $\psi(x)$ in the regions $0 < x < L$ and $x > L$. Use the following basic form:

$$\psi(x) = \begin{cases} A(\text{mess}) + B(\text{other mess}) & 0 < x < L \\ C(\text{third mess}) + D(\text{fourth mess}) & x > L \end{cases}$$

   b) Use the condition as $\psi(x \to \infty)$ and continuity at $x = 0$ and $x = L$ to rewrite your answer to part (a) with only 1 of A, B, C, and D remaining. (Please use the shorthand $k = \sqrt{\frac{2mE}{\hbar^2}}$ and $\kappa = \sqrt{\frac{2m(U_\circ - E)}{\hbar^2}}$ in your answer).

4. In class, I told you the 3rd excited state solution of the time-independent wave-function for a particle in a simple harmonic potential of the form $U(x) = \frac{1}{2}m\omega^2 x^2$ was equal to:

$$\psi_3(x) = A_3(3u - 2u^3)e^{\frac{-u^2}{2}}$$

with A_3 a normalization constant and $u = x\sqrt{\frac{m\omega}{\hbar}}$.

a) Show (via explicit substitution) that this is a solution to the Schrödinger equation for this potential with an eigenvalue $E = \frac{7}{2}\hbar\omega$.

b) Find A_3 by integrating $\psi^*\psi$ from $-\infty$ to ∞ and equating the integral to 1. (You may wish to use integral tables and/or a computer algebra system like Mathematica to help with this. That's perfectly ok – but make sure you cite your sources and/or include any code you wrote!)

Mathematica and/or MATLAB Problem

Please *PRINT OUT YOUR SOLUTIONS* to this problem, *put your name on them*, and turn them in *stapled but separately from the rest of the assignment.* Also, email me your code to `LarsenML@cofc.edu`.

5. The normalized eigenfunction solutions to the infinite square well potential are as follows:

$$\psi_n(x) = \begin{cases} \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right) & 0 \le x \le L \\ 0 & x < 0 \text{ or } x > L \end{cases}$$

with the energies associated with each state equal to:

$$E_n = \frac{n^2\hbar^2\pi^2}{2mL^2}$$

a) Build a MATLAB or Mathematica function that takes in m, L, and n and outputs the energy associated with the level *in Electron Volts!*

b) Use MATLAB or Mathematica somehow to find what L must be for E_1 for an electron to be 13.6 eV.

c) Use MATLAB or Mathematica to generate a function/interactive plot that allows the user (through the *Manipulate* command or via an argument to a function) to plot $\psi_n(x)$ or $\psi_n^*(x)\psi_n(x)$ (the user gets the choice). Of course, n must be an integer. You may set $L = 1$ for convenience if you wish.