Assignment X, PHYS 230 (Introduction to Modern Physics)
Spring 2017
Due Wednesday, 4/12/17 at start of class

1. A particle at some time has the following wave equation:

$$\psi(x) = \begin{cases} 0 & x < 0 \\ Ae^{-kx} & x \geq 0 \end{cases}$$

   with k and A unspecified (real and nonzero) constants.

   a) This $\psi(x)$ is not a valid solution for any real potential-energy function. How can you tell?

   b) If we ignore the issue pointed out in part (a), we can still treat this as a normal wave-function. What must A be if the wave-function is properly normalized?

   c) Given your answer to part (b), what is the probability that the particle is between $x = \frac{1}{k}$ and $x = \frac{3}{k}$?

2. In a region of space, a particle has a wave function given by $\psi(x) = Ae^{\frac{-x^2}{2L^2}}$ and has energy $E = \frac{\hbar^2}{2mL^2}$ where L is some length. Find the potential energy as a function of x and sketch (i.e. graph by hand) $U(x)$ vs. x.

3. We've talked in class some about normalizing wave functions. I also alluded to the fact that once you know the wave function for a system, you can find out a bunch of other information. The most straightforward of these is what is called the "expectation value" of an operator. This problem will get you working towards that idea. As you know, to find the probability that a particle will be between points a and b, you merely have to calculate $\int_a^b \psi^*(x)\psi(x)\mathrm{d}x$ (assuming $\psi(x)$ is a normalized wave-function). Quite often, another quantity that is desired to be known is the "expected value of the position" $\langle x \rangle$. This can be computed by calculating $\langle x \rangle = \int_{-\infty}^{\infty} \psi^*(x)x\psi(x)\mathrm{d}x$. Again, $\psi(x)$ must be a normalized wave-function.

   For the wave functions associated with an infinite square well of width L between 0 and L (e.g.:)

$$\psi(x) = \begin{cases} \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right) & 0 < x < L \qquad n = 1, 2, 3, \ldots \\ 0 & \textit{otherwise} \end{cases}$$

   calculate:

   a) $\langle x \rangle$ (your answer might depend on n).

   b) $\langle x^2 \rangle = \int_{-\infty}^{\infty} \psi^*(x)x^2\psi(x)\mathrm{d}x$ (your answer might depend on n).

4. Let us define the following wave function:

$$\psi(x) = \begin{cases} 0 & x < -L \\ (C - Ax) & -L < x < 0 \\ D & 0 < x < L \\ G + Hx & L < x < 2L \\ 0 & x > 2L \end{cases}$$

with A, C, D, G and H unknown constants and L some specified length. (Note that this isn't actually a valid wave-function, since the derivative of $\psi(x)$ is not continuous at $-L$, 0, L, or $2L$. However we are not going to worry about that).

a) Use continuity of ψ at boundaries to write the above wave function in terms of just D, L, and x.

b) Use the normalization condition ($\int_{-\infty}^{\infty} |\psi(x)|^2 \mathrm{d}x = 1$) to determine what D must be in terms of L.

c) Use the above answers to determine the probability of the particle being between $\frac{L}{2}$ and L.

Extra credit Mathematica and/or MATLAB Problem

If you do this problem, please PRINT OUT YOUR SOLUTIONS to this problem, put your name on them, and turn them in stapled to the rest of this assignment. Also, email me your code to `LarsenML@cofc.edu`.

5. The normalized eigenfunction solutions to the infinite square well potential are as follows:

$$\psi_n(x) = \begin{cases} \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right) & 0 \le x \le L \\ 0 & x < 0 \text{ or } x > L \end{cases}$$

with the energies associated with each state equal to:

$$E_n = \frac{n^2 \hbar^2 \pi^2}{2mL^2}$$

a) Build a MATLAB or Mathematica function that takes in m, L, and n and outputs the energy associated with the level *in Electron Volts!*

b) Use MATLAB or Mathematica somehow to find what L must be for E_1 for an electron to be 13.6 eV.

c) Use MATLAB or Mathematica to generate a function/interactive plot that allows the user (through the *Manipulate* command or via an argument to a function) to plot $\psi_n(x)$ or $\psi_n^*(x)\psi_n(x)$ (the user gets the choice). Of course, n must be an integer. You may set $L = 1$ for convenience if you wish.