Assignment X, PHYS 272 (MAP) [Last one! Hooray!]
Fall 2014
Due 11/21/14 at start of class

> I'm not sure exactly when we're going to get to this material. Fortunately, the questions are all pretty understandable even if you haven't done formal coursework in statistics and/or probability. You may have to look a few things up, but you should be ok as long as you start in advance. If you have difficulties, stop by for office hours or, if I'm not on campus, send me an email and I'll help to the degree I can.

1. Show that if A and B are independent:

$$P(A) = \left(\frac{P(A \cup B) - P(B)}{1 - P(B)}\right)$$

2. Use the above result to find the probability of A if the probability of B is 0.4 and the probability $p(A \cup B) = 0.6$.

3. Let's say thing A happens with probability 0.4, thing B happens with probability 0.55, and thing C happens with probability 0.7. Let's also so that the probability that both A and B happens ($p(AB)$) = 0.3 and the probability that (either/both A and B happen) and (C happens) $[p((A \cup B)C)] = 0.5$. What is the probability that none of A, B, or C happens?

4. There are two die. One that is fair (1/6 chance of coming up 1,2,3,4,5, or 6) and one that is "loaded" in such a way that the probability of rolling a 1 is 50% and the chances of rolling the other 5 numbers is each 10%. You can't remember which die is which.

   a) You take one of the die and roll it and it comes up 4. What is the probability you have rolled the fair die?

   b) You take one of the die and roll it three times and you get 2,3,1. What is the probability you have rolled the fair die?

   c) You take one of the die and roll it four times and it comes up 1,1,2,1. What is the probability you have rolled the fair die?

5. A continuous probability density function is defined as follows:

$$f(x)\mathrm{d}x = \begin{cases} \alpha x^2 \mathrm{d}x & a < x < b \\ 0 & \text{otherwise} \end{cases}$$

   a) Find α if this is a well-defined probability density function.

   b) Given the value of α found in part (a), find μ for this probability density function.

6. Quite possibly the most important probability distribution in experimental science is the Poisson distribution. (You could make a pretty strong argument for the Gaussian as well). The probability of observing k events for a random process exhibiting Poisson statistics is equal to:

$$p_\mu(k) = \frac{\mu^k \exp(-\mu)}{k!}$$

where μ is the mean number of events (or, if you prefer, the expectation value of the number of events).

   a) Clearly show that this is a legitimate probability distribution. (In other words, show $\sum_{k=0}^{\infty} p_\mu(k) = 1$).
   b) What is the probability of observing 0 events?
   c) Approximately what is the probability of observing 1 event if $\mu \to 0$? (i.e. assume μ is small, and μ^2 is negligibly small).

> We quite possibly will not have talked about the content in these last two questions in class before this homework is due. This means you might have to google this stuff, or find a written resource to help you. These are important concepts, however, and I think they are worth your time.

7. This problem is related to the central limit theorem.

   a) Briefly summarize the central limit theorem.
   b) What conditions must be met for the central limit theorem to apply?

8. This problem is related to Chebyshev's inequality.

   a) State / summarize Chebyshev's inequality.
   b) According to Chebyshev's inequality, what can we definitively say about the probability of a random variable x being more than 3σ from μ?
   c) If a variable is distributed according to Gaussian statistics, you expect 99% of measurements to be within 3σ of the mean. This is not at odds with Chebyshev's inequality. Explain why not.