Assignment X, PHYS 301 (Classical Mechanics)
Spring 2014
Due 4/4/14 at start of class

1. Do problem 7.44 in your text. (Problem 6 on HW VIII was essentially problem 7.29).

2. Similar to the above, replot $\phi(t)$ with the new conditions with $g = \ell = 1$ but $\omega = 3$ and $R = 2$. Let initial conditions be $\phi = 2$ and $\dot{\phi} = 0$. Plot your solution for $0 < t < 10$. Comment on your plot.

3. Similar to the above, replot $\phi(t)$ with the new conditions with $g = \ell = 1$ but $\omega = 0.7$ and $R = 2$. Let initial conditions be $\phi = 0.1$ and $\dot{\phi} = -0.9$. Plot your solution for $0 < t < 100$. Comment on your plot.

4. A particle of mass m moves in the field of a fixed force center at the origin from which it is repelled with a force of magnitude $\frac{m\beta}{r^3}$ where β is a positive constant. At very large distances from the origin the particle is moving with velocity $v_\circ$ which – if the particle were not deflected – would carry it along a straight line with closest approach to the origin being b (this distance is often called the "impact parameter"). Find the distance of actual closest approach a given the presence of the force at the origin in terms of b, β, and $v_\circ$.

5. The orbit of a particle moving in a central field is a circle passing through the origin $r = r_\circ \cos\theta$. The force law is of the form $F(r) \propto r^n$. Find n. (It is an integer).

6. A particle moves in a spiral orbit given by $r = a\theta$ (with a a constant). If $\theta = kt^1$ (with k constant), is the force a central force? If so – how do you know? If not, then $\theta = kt^\gamma$ (with k constant and $\gamma \neq 1$) *DOES* correspond to a central force for some value of γ. Find the value of γ.

7. A particle moving in a central field describes the spiral orbit $r = r_\circ e^{k\theta}$ with k and $r_\circ$ constants.

   a) The force law is of the form $F(r) \propto r^n$. Find n.

   b) θ depends logarithmically on time. Find the exact expression for $\theta(t)$, assuming $\theta(0) = \theta_\circ$.

8. A comet moves in a parabolic orbit lying in the plane of the Earth's orbit. Regarding the Earth's orbit as circular and of radius a, show that the points where the comet intersects the Earth's orbit are given by:

$$\theta = \pm \cos^{-1}\left(\frac{2p}{a} - 1\right)$$

   where p is the perihelion distance of the comet defined at $\theta = 0$.