Assignment XI (Extra Credit), PHYS 230 (Introduction to Modern Physics)
Fall 2015
Due Tuesday, 12/1/15 at start of class

1. A free electron with kinetic energy 10 eV is incident on a barrier potential of 20 eV and width $1\AA$. What is the probability that the electron tunnels through the barrier to the other side (e.g. what is the transmission probability?)

2. A free electron with kinetic energy 10 eV is incident on a barrier potential of 100 eV and width $1\AA$. What is the probability that the electron tunnels through the barrier to the other side (e.g. what is the transmission probability?)

3. A baseball (mass $\approx$ .15 kg, diameter of about 9 cm) is in a bucket with walls approximately 25 cm tall. Let us say the width of the wall of the bucket is 1 mm, and the baseball has kinetic energy of at most 7.5 μJ. What would be the probability of the baseball tunneling out of the bucket when it reaches the wall? (Assume that a barrier potential is appropriate here). This isn't all that straightforward – you have to work out some geometrical issues and there's more than one way to do so, so please make sure to include enough text that I clearly understand your line of reasoning. (This question isn't exactly well-posed, but I want to see what you do with it. Make sure your answer passes a "sanity check".)

4. Recall the Rydberg-Ritz formula (as constructed with insight from the Bohr model):

$$\frac{1}{\lambda} = \frac{E_\circ}{hc}\left(\frac{1}{n^2} - \frac{1}{m^2}\right)$$

Show that the shortest wavelength emitted by a single electron in a one-electron atom moving from an excited state to a final state n can be written as:

$$\lambda_{\text{min}} = \frac{4\pi\hbar n^2}{cm_e Z^2 \alpha^2}$$

(You may ignore the reduced mass correction.) (You may have to look up α; it is the so-called "fine structure constant").

5. In class notes, Dr. Larsen started with the Bohr postulates and derived the following for the velocity, energy, and radius of the ground state of a single electron atom:

$$v_\circ = \frac{Ze^2}{4\pi\epsilon_\circ\hbar}$$

$$E_\circ = \frac{-m}{2}\left(\frac{Ze^2}{4\pi\epsilon_\circ\hbar}\right)^2$$

$$r_\circ = a_\circ = \frac{4\pi\epsilon_\circ\hbar^2}{Ze^2 m}$$

Note that this notation is a little weird, because $v_\circ$ is the velocity for the $n = 1$ state, so $v_\circ$ and v_n with $n = 1$ would give you the same thing. Use the same basic procedure that Dr. Larsen used for the ground state to find expressions for v_n, E_n, and r_n for any state of a single electron atom. (As a simple check, your expressions should reduce to the expressions for $v_\circ$, $E_\circ$, and $r_\circ$ when $n = 1$). (You may wish to use the results from this problem in some of the problems below).

6. For a Hydrogen atom in the Bohr model:

   a) What would the velocity of a ground state electron be (I'm looking for a numerical answer here)?

   b) What would the velocity of an electron in the $n = 12$ excited state be?

   c) Since the velocity of an electron is clearly highest in the ground state (if you didn't get that, your answer to the previous problem is likely incorrect), what is the value of γ (from relativity) for a ground state electron in Hydrogen? Use your computed value to comment on the need to introduce relativistic corrections into Bohr's model.

   d) A hydrogen atom exists in an excited state for typically around 10 nanoseconds. How many revolutions would Bohr's theory suggest that an electron would make in an $n = 3$ state before returning to the ground state?

7. In a Lithium atom ($Z = 3$), two electrons are in the $n = 1$ orbit and the third is in the $n = 2$ orbit. (Only two are allowed in the $n = 1$ orbit because of the exclusion principle, which will be discussed later in this class or, failing that, in Quantum). The interaction of the inner electrons with the outer one can be approximated by writing the energy of the outer electron as:

$$E = -(Z')^2 \frac{E_1}{n^2}$$

where $E_1 = 13.6$ eV, $n = 2$, and Z' is the effective nuclear charge, which is less than 3 because of the "screening" effect of the two inner electrons. (Since there are negative charges between the outer electron and the nucleus at least part of the time, the outer electron doesn't feel the total net effect of the full nuclear charge). Using the measured ionization energy of 5.39 eV needed to remove the outermost electron, calculate Z' for this atom.

8. The transition from the first excited state to the ground state in Potassium results in the emission of a photon with $\lambda = 770$nm. If Potassium vapor is used in a Franck-Hertz experiment, at what voltage would you expect to see the first decrease in current?