Assignment XI, PHYS 301 (Classical Mechanics)
Spring 2014
Due 4/11/14 at start of class

The last couple of problems on this assignment actually have some numbers. You may use calculators or other computational aids to assist you in coming up with numerical solutions to these problems. However, you should do any integrals/derivatives/etc. yourself.

1. A rotating tabletop (merry-go-round) on Earth of radius R rotates with constant angular speed Ω. A deck of cards (in the box) sits on the merry-go-round. The coefficient of static friction between the merry-go-round and the deck of cards is μ_s. What is the maximal distance the deck of cards can be from the center of the merry-go-round without starting to slide off? (Note; you may neglect the Earth's rotation).

2. Similar to the above, we have a rotating tabletop (merry-go-round) of radius R. Now the angular speed of the merry-go-round obeys the relationship $\Omega(t) = \alpha t$ with α some (positive) constant. The deck of cards is placed at distance $\frac{R}{3}$ from the center. (The coefficient of static friction is still μ_s, and you may still neglect the rotation of Earth). At what time does the box of cards start to slip? (This is reasonably ugly; please try to clean it up as much as possible. No fractions within fractions! And put everything over a common denominator in your final answer).

3. In class, we have shown (or will show) that, due to the rotation of the Earth and the Coriolis force, a mass dropped at rest above a point on the Earth's surface ends up landing East of its initial drop point. (This makes perfect sense if you think of the Earth rotating beneath the item as it falls). A less intuitive result is that the Coriolis force also has an influence on moving the mass North or South of its initial drop point. (This is an *exceedingly* minor effect – about a million times weaker than the East/West effect – but it is there, and it has been measured). I'm not going to ask you to calculate the magnitude of this deflection; there are three forces that influence it (variation of g with height, the Centrifugal force, and the Coriolis force all play a role, and they contribute nearly the same amount to the effect). The end result is that each of these 3 forces push the object in the same direction. Is the mass deflected North or South if dropped in the Northern hemisphere of Earth? Hint: In class, we used a first-order approximation to attack the coupled differential equations for $\ddot{x}$, $\ddot{y}$, and $\ddot{z}$. If you use the first-order result for $x(t)$ and substitute it back in, you should get some nonzero quantity for $y(t)$. All you are concerned with is figuring out the sign and how that maps back to a North/South motion.

4. A smooth (frictionless) rod of length ℓ rotates in a plane with constant angular velocity ω about an axis fixed at the end of the rod and perpendicular to the plane of rotation. A bead of mass m is initially positioned at the stationary end of the rod and given a slight push such that its initial speed directed down the rod is $v_\circ = \omega\ell$. Calculate how long it takes for the bead to reach the other end of the rod.

5. An ant crawls with constant speed in a circular path of radius R on a turntable rotating with constant angular speed Ω. The circular path is concentric with the center of the turntable. If the mass of the ant is m and the coefficient of static friction with the surface of the turntable is μ_s, how fast – relative to the turntable – can the ant crawl before it starts to slip if it travels:

   a) In the direction of rotation? (Solve this part of this problem in an inertial reference frame. Note – this is , by most accounts, the easier choice).

   b) Opposite to the direction of rotation? (Solve this part of this problem in the rotating reference frame. This is harder for most people).

   Note: if you like, you can check your answers by solving each part in the other reference frame as well. You should get the same answer once you account for the relative motion between the frames.

6. In addition to the Coriolis and Centrifugal force, there is a third "fictitious" force associated with a rotating system – sometimes called the "azimuthal force" or "Euler force". This force has magnitude $|m\vec{r} \times \dot{\vec{\Omega}}|$. Find the ratio of this Azimuthal force to the Coriolis force on Earth. Assume you are traveling in a car moving 30 m/s to the North at a latitude of 45 degrees North. You may assume the Earth is spherical. To help you estimate $\dot{\vec{\Omega}}$, note that the axial tilt of the earth changes from about 22.1° to about 24.5° and back periodically. It takes about 40,000 years for a full cycle. For simplicity, assume that the change in angle occurs at a constant rate over the 20,000 years that it takes to change the 2.4 degrees.

7. A cannonball is fired straight up with initial speed $v_\circ$.

   a) Assuming g is constant and neglecting air resistance, show that the cannonball will hit the ground displaced from the initial points of upward motion by an amount:

   $$\frac{4\Omega v_\circ^3 \cos\theta}{3g^2}$$

   where θ is the latitude and Ω is the Earth's angular speed.

   b) What direction will the largest component of the displacement be? (North, South, East, or West?) (You may assume $\theta > 0$ – in other words, you are in the Northern Hemisphere).

   c) Let's see what this means in terms of a realistic system. Look up the latitude for Charleston. Let's say you launch a 10 inch cannonball straight up on the battery with velocity of 1832 feet per second. (I know the units suck. Blame the military). How far is this displaced from the launch point upon landing? (Assume no air resistance).