Homework 12, PHYS 415 (Fluid Mechanics)
Spring 2019
Due Thursday 18 April 2019 at Beginning of Class
(LAST HOMEWORK! WOO-HOO!)

As always, turn in your legible and annotated work on separate paper.

1. Some (most? all?) of you may know that I spent the 2017-2018 academic year on Sabbatical at Michigan Technological University. Part of the reason I went there was to work with their Π-chamber; a cloud chamber that induces turbulent air motion inside a closed box through Rayleigh-Bénard convection. This problem relates to this real physical system. You can read more about the chamber at http://phy.sites.mtu.edu/cloudchamber/, though I think I give you all the information you will need below.

   Following standard Rayleigh-Bénard configuration, the cloud chamber encloses a volume that is 1 meter high with a square cross-sectional base of 2 m x 2 m. (We will assume that the cylindrical inset is outside of the chamber, so the internal volume is just 4 cubic meters). The side walls of the chamber will be neglected for this problem, and we will design the system to be unstable with the ceiling of the chamber at a temperature T_C and the floor of the chamber at a temperature T_H with $T_H - T_C = \Delta T$. Inside the chamber we will assume is dry air.

   a) According to your text, the critical Rayleigh number required for the onset of the instability the results in convective cells within the chamber is 1708. Given the dimensions of the Π chamber, what ΔT would be necessary to induce convection in the chamber?
   b) Assuming that the maximum horizontal velocity in this system is 12.5 μm/s for $\Delta T = 43\ \mu$K, what would be the maximum horizontal velocity for $\Delta T = 2.35$ K if the Landau model is correct for this system and we are still close enough to Ra_C so that the scaling suggested by the model holds?
   c) Your answer to part (a) will be much smaller than the ΔT values typically used to generate clouds within the chamber. A typical cloud inside the chamber is induced with (moist) air having $\Delta T \approx 10$ K. Ignoring any influence associated with the different composition of the moist air, what is the Rayleigh number inside the chamber under these cloud-generating conditions?

2. Watch the "flow-instabilities" fluid mechanics films video at:
   https://techtv.mit.edu/collections/ifluids/videos/32605-flow-instabilities
   and answer the following questions.

   a) Does the wind-speed where a clear flow instability "takes hold" in the perturbed wave demonstration (just under halfway through the movie) depend on the frequency of the perturbation?
   b) What are the two forces that act to decrease the onset of the instability in the growing wave demonstration (about halfway through the movie)?
   c) Based on the formation of the Bénard cells seen at about the 21 minute mark, estimate the depth of the fluid in the pan.

(MORE ON BACK)

3. Based on the formula in your text for the onset of the Taylor-Couette instability, what is the *frequency of rotation* for an inner cylinder of radius $r_1 = 14$ cm with an outer cylinder of radius $r_2 = 15cm$ if the fluid between them is:

   a) Air at 20° C
   b) Water at 20° C
   c) Honey (Use 37.8° C for this one; it is easier to find).
   d) Glycerine (Glycerol) at 20°C

4. If you have two infinitely deep fluids stacked on top of each other (don't think about that too hard), the dispersion relationship due to the Kelvin-Helmholtz instability takes the following form:

$$\omega^2 = gk\left(\frac{\rho_2 - \rho_1}{\rho_2 + \rho_1}\right).$$

   What are the phase and group velocities for these waves?

5. Look up Squire's theorem and use it to justify the claim that you merely have to examine two-dimensional perturbations to determine if a fluid flow might lead to an instability at a particular Reynolds number.

6. I hope this has been a productive class for you to take this semester. As you know, I don't have much of a formal fluids background – but I've done the best I can in putting together lectures and homework sets that will teach you some of the important ideas associated with the topic. Are there any suggestions you have for me if I am ever asked to teach this course again?