## Assignment, PHYS 230 - Due on Thursday, January 26 at start of class

Read Chapter 1 of your text.

1. We'll start out with something rather straightforward. See figure 1 (shamelessly cribbed from your text). If $v$ is steady at $5 \mathrm{~m} / \mathrm{s}$, each boat always travels with a speed (relative to the water) of $13 \mathrm{~m} / \mathrm{s}$ and $L$ is 3.5 km . [Hint: look at your textbook if you need help; the problem where I grabbed the figure from gives lots of help.]
a) How long does it take boat 1 to return to point A ?
b) How long does it take boat 2 to return to point A?
c) Which boat is faster, and by how much?


Figure 1: The figure associated with problem 1.
2. Your instructor skipped some steps in getting to the final Lorentz transformation (in the spherical light wave approach). Given his intermediate step:

$$
x^{2}-c^{2} t^{2}=[\lambda(x-v t)]^{2}-c^{2}\left[\frac{\left(1-\lambda^{2}\right) x}{\lambda v}+\lambda t\right]^{2}
$$

equate coefficients of the $t^{2}$ term to show

$$
\lambda=\frac{1}{\sqrt{1-\frac{v^{2}}{c^{2}}}}
$$

[hint, you're going to have to FOIL and then do some algebra].
3. Usually, we write the quantity in the above question as $\gamma$. Use MATLAB, Mathematica, Excel (if you must), or hand-draw a plot of $\gamma=\left(1-\frac{v^{2}}{c^{2}}\right)^{-1 / 2}$ as a function of $v / c$ for $0 \leq v / c \leq 1$. Take care to note any asymptotes, and make sure you get any $x$ or $y$ intercepts right. If you would like to get help on how to do this with a computational tool and don't know how to do it yet, come visit me in office hours. (Later on this semester, you won't have the opportunity to graph things like this by hand - but you will get some instruction on how to use MATLAB and/or Mathematica to do this sort of thing for you).
4. The proper length of one spaceship is three times that of another. The two spaceships are traveling in the same direction and, while both are passing overhead, an Earth observer measures the two spaceships to have the same length. If the slower spaceship is moving with a speed of $0.35 c$, determine the speed of the faster spaceship.
5. When at rest, the $\Sigma^{-}$particle has a lifetime of 0.15 ns before it decays into a neutron and a pion. One particle $\Sigma^{-}$particle is observed to travel 3.0 cm in the lab before decaying. What was its speed. (Hint: Its speed was not $\frac{2}{3} c$ ).
6. A stick of length $L_{\circ}$ is at rest in $S$, making an angle $\theta$ with the $x$-axis. Show that for an observer in the $S^{\prime}$ frame:
a) the length is measured to be

$$
L^{\prime}=L_{0}\left(\cos ^{2} \theta / \gamma^{2}+\sin ^{2} \theta\right)^{1 / 2}
$$

b) the angle with respect to the $x^{\prime}$-axis is measured to be

$$
\tan \theta^{\prime}=\gamma \tan \theta
$$

7. A Physics professor on Earth gives an exam to his students who are on a spaceship traveling at speed $v$ relative to the Earth. The moment the ship passes the professor, he signals the start of the exam. If he wishes his students to have time $T_{\circ}$ (spaceship time) to complete the exam, show that he should wait a time (Earth time) of:

$$
T=T_{\circ} \sqrt{\frac{1-\frac{v}{c}}{1+\frac{v}{c}}}
$$

before sending a light signal telling them to stop. Hint: Remember that it takes some time for the second light signal to travel from the professor to the students.
8. Show that if $0<v_{1}<c$ and $0<v_{2}<c$ are two velocities pointing in the same direction, the relativistic sum of these velocities, $v$, is greater than $v_{!}$and greater than $v_{2}$ but less than $c$. (This means prove it; don't just plug in one particular case).

