Assignment I, PHYS 409 (Electromagnetism I)
Fall 2019 Due 8/27/19 at start of class

In the beginning of this course, we're going to do a little review of vector calculus. This *should* all be review, since this course has DiffEq as a prerequisite, and a prerequisite for DiffEq is Calc III. However, you may understandably be rusty on some of this. This homework assignment should serve as a quick refresher.

Some general comments about homework assignments in this class. (Please read the following carefully; there won't be any warnings).

As always, working with classmates is fine (even encouraged), but the work you turn in must be your own! Let me explain this clearly once and for all, since it is often misunderstood by students.

- If you work with a classmate on a problem, the first thing you need to write on your solution to that problem is to indicate which classmates you worked on the problem with.
- Although figuring out a method of attack with a classmate is a good idea, I expect each of you to work through the underlying math on your own. If I see the exact same simple math errors on multiple different student homework assignments, I'll know that you aren't likely all pushing through the math on your own – and I'll take off more points for the error on both/all of your papers.

Please put your answers on a separate sheet of paper. If you want to type them up in LaTeX, that would be awesome – but not expected. However, please show all work and explain all steps. The more you describe to me, the more partial credit I can justify giving you if you make a mistake!

I expect to be turning around these assignments in time to return them on Thursday, so – since time is limited for me – please keep your work legible, clearly mark your final answers, and include enough words that I can understand your thought process in case you took a wrong turn somewhere. A long string of equations that begins from an invalid starting point isn't going to earn much credit! Also, sometimes I can't figure out how you got from step n to step (n+1); maybe it was brilliant – maybe it was just BS because you couldn't figure how to make that leap, either. If you don't have an explanation and I can't see the link, I can't give you credit – and I'm not that smart, so you're better off leading me by the hand. This goes double on "show that the following expression is true" type questions; I KNOW where you are trying to get (so do you), so there's a great temptation to just try to hide the reasoning through nonsense if you get stuck. If I can't follow it, you won't earn credit.

Unless stated otherwise, use of technology (Mathematica, MATLAB, calculators, on-line integration tools, etc.) is not allowed. It is fine to use these to check your work, but a solution that says something like "and this integrates to this because Mathematica says so" won't fly.

Fractions within fractions are evil. Do not leave your answer in a form that has them. Additionally, don't mix decimal numbers with fractions (without calculators, there really shouldn't be numbers like 0.372 in your problem anyway). Unless stated otherwise, leave π, e, $\epsilon_\circ$, etc. as symbols rather than approximating them numerically. Always take Taylor expansions out to at least one more term than you think you need.

This is just a start; throughout the semester I will likely add additional things that won't fly – but for now, this is enough before we actually get to the real homework questions. Good luck and, if there is stuff on this assignment you've never seen before, make sure to catch up ASAP or this will be a rough semester.

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Start by reading chapter 1 (either edition); almost all of this material should be review.
For the following problems, let us define the following:

$$\vec{A} = 3\hat{x} - 5\hat{y} + 2\hat{z} \qquad \vec{B} = 2\hat{x} + \hat{y} \qquad \vec{C} = 7\hat{x} + 2\hat{y} - 4\hat{z}$$
$$\vec{D} = x\hat{x} - 2z\hat{y} + (3xz + 2)\hat{z} \qquad E = 3x^2 y e^{2z} \qquad F = 4\sin(xyz)$$

1. Verify the distributive law holds for dot products by calculating $\vec{A} \cdot (\vec{C} - \vec{B})$.
   a) First calculate $\vec{C} - \vec{B}$, then take the dot product of $\vec{A}$ with the difference.
   b) Calculate $\vec{A} \cdot \vec{C}$ and add it to $\vec{A} \cdot (\vec{-B})$. (Should be the same numerical answer as part (a)).
2. Calculate:
   a) $\vec{A} \times \vec{B}$
   b) $\vec{B} \times \vec{C}$
   c) $(\vec{A} \times \vec{B}) \times (\vec{B} \times \vec{C})$
   d) $(\vec{A} \times \vec{B}) \times \vec{B}$
   e) $(\vec{A} \times \vec{B}) \cdot \vec{A}$
   f) Let $\vec{Z} = \vec{A} \times \vec{B}$ (computed in part (a)). Find $\hat{Z}$ (a unit vector in the Z direction).
3. Calculate:
   a) $\vec{\nabla} E$
   b) $\vec{\nabla} F$
   c) $\vec{\nabla} \cdot \vec{A}$
   d) $\vec{\nabla} \cdot (E\vec{A})$
   e) $\vec{\nabla} \cdot \vec{D}$
   f) $\vec{\nabla} \times \vec{D}$
   g) $\nabla^2 E$ [note; if you haven't seen ∇^2 before, it is called the "Laplacian"; $\nabla^2 E = \vec{\nabla} \cdot \left(\vec{\nabla} E\right)$].
   (Please indicate on your homework if you needed this hint or not; it won't affect your grade. I just would like to know how often they talk about the Laplacian in math classes).

4. Let $\Phi(x,y,z) = 4x^2 yz\cos(3x) + 5x$
   a) Find $\vec{\nabla}\Phi$.
   b) Find $\vec{\nabla} \times \left(\vec{\nabla}\Phi\right)$.
   c) Find $\vec{\nabla} \cdot (\vec{\nabla}\Phi)$ (a.k.a. $\vec{\nabla}^2\Phi$).
5. Show (in general) that $\vec{\nabla} \cdot (\vec{\nabla} \times \vec{I}) = 0$ for any vector field $\vec{I}$. (Note that this requires more than just making up some vector $\vec{I}$ and showing that it is true for that vector.)
6. Calculate the following in spherical coordinates (what your book calls spherical polar coordinates). Note – there are formulas in the front cover of your text that may come in handy.
   a) $\vec{\nabla} \cdot \left(r^2 \hat{r}\right)$
   b) $\vec{\nabla} \cdot (r\hat{r})$
   c) $\vec{\nabla} \cdot \left(\frac{1}{r}\hat{r}\right)$
   d) $\vec{\nabla} \cdot \left(\frac{1}{r^2}\hat{r}\right)$
   e) $\vec{\nabla} \cdot \left(\frac{1}{r^3}\hat{r}\right)$
   f) $\nabla^2 \left(r^2 \hat{r}\right)$
   g) $\nabla^2 (r\hat{r})$
   h) $\nabla^2 \left(\frac{1}{r}\hat{r}\right)$
   i) $\nabla^2 \left(\frac{1}{r^2}\hat{r}\right)$
   j) $\nabla^2 \left(\frac{1}{r^3}\hat{r}\right)$
7. Convert between coordinate systems as indicated:
   a) $3\hat{x} - 4\hat{y} + 2\hat{z}$ to cylindrical
   b) $-2\hat{x} + 3\hat{y} - 5\hat{z}$ to spherical-polar
   c) $3\hat{s} + (\pi/3)\hat{\phi} + 2\hat{z}$ to cartesian
   d) $2\hat{s} + (3\pi/4)\hat{\phi} - 4\hat{z}$ to spherical-polar
   e) $1\hat{r} + (\pi/4)\hat{\theta} + (11\pi/6)\hat{\phi}$ to cartesian
   f) $7\hat{r} + (\pi/6)\hat{\theta} + (4\pi/3)\hat{\phi}$ to cylindrical
8. Find a simple expression for $f(n)$ when $f(n) \equiv \vec{\nabla} \times r^n \hat{r}$.