**Assignment II, PHYS 409 (Electromagnetism I)**
**Fall 2019 Due 9/3/19 at start of class**

Please supply your complete, legible, and well organized solutions on separate paper. No Mathematica, calculators, or other technological aids should be used (or are necessary) to complete this homework assignment.

1. Time for a little practice/review with multiple integrals in a non-cartesian coordinate system.

   a) A circular cylinder of radius $R$ runs co-axial with the $z$ axis between $z = 0$ and $z = H$. The mass density inside this cylinder can be written as:

   $$\rho(s,\phi,z) = \begin{cases} \frac{\rho_\circ s}{R}(1+\alpha z) & 0 \le s \le R, 0 \le z \le H \\ 0 & \text{otherwise} \end{cases}$$

   with $\rho_\circ$ and $\alpha$ positive real constants. What is the total mass of the cylinder in terms of $\rho_\circ$, $\alpha$, $R$, and/or $H$?

   b) Consider the vector field $\vec{F} = 3\beta s^2 \hat{s} + 2e^{\gamma z}\hat{z}$ with $\beta$ and $\gamma$ positive real constants. Find $\oiint \vec{F}\cdot \mathrm{d}\vec{a}$ for the same cylindrical region specified in part (a) above. Note: DO NOT USE THE DIVERGENCE THEOREM TO REWRITE THIS AS A TRIPLE INTEGRAL; I want to make sure you can do these double integrals in cylindrical coordinates. Of course you can double-check your answer by doing the triple integral on your own, but don't turn it in.

   c) Consider a sphere of radius $R$ at the origin. From this sphere, we are only going to consider the portion $0 \le \theta \le \Omega$ with $\Omega$ some constant between 0 and $\pi$. The mass density inside this spherical piece can be written as:

   $$\rho(r,\theta,\phi) = \begin{cases} \frac{\rho_\circ(1+\sin(\phi))r}{R} & 0 \le r \le R, 0 \le \theta \le \Omega \\ 0 & \text{otherwise} \end{cases}$$

   with $\rho_\circ$ a positive real constant. What is the total mass of the piece of the sphere in terms of $\rho_\circ$, $R$, and $\Omega$?

2. We're going to test the divergence theorem for a single case by evaluating both sides of the equality and verifying they match. Calculate the following for vector field $\vec{A} = \vec{r} = x\hat{x} + y\hat{y} + z\hat{z}$ over the volume bounded by the sphere $S$ defined by $x^2 + y^2 + z^2 = 9$. (Don't use the divergence theorem; we're testing it to see if our answers to parts (a) and (b) below are the same).

   a) $\iiint_S \left(\vec{\nabla}\cdot\vec{A}\right)\mathrm{d}\tau$
   b) $\oiint_{\partial S} \vec{A}\cdot\mathrm{d}\vec{a}$

3. Evaluate the following integral: $\int_{-\infty}^{\infty} 2x^3\delta(4x-2)\,\mathrm{d}x$.

4. Evaluate the following integral: $\int_{-1}^{1} (\cosh x)\,\delta(-2x)\,\mathrm{d}x$

5. Let $\vec{\Lambda} = 4y\hat{x} + x\hat{y} - 3z\hat{z}$. Find $\iint (\vec{\nabla}\times\vec{\Lambda})\cdot\mathrm{d}\vec{a}$ over the hemisphere $x^2 + y^2 + z^2 = a^2$ with $z \ge 0$.

MORE ON BACK!

6. Determine if the following vector field $\Omega$ is conservative.

$$\vec{\Omega} = (3x^2yz - 3y)\hat{x} + (x^3z - 3x)\hat{y} + (x^3y + 2z)\hat{z}$$

7. For the vector field in the above problem, find the total work done in moving the object from the origin to the point $2\hat{x} - 3\hat{y} + 5\hat{z}$. If the above vector field was conservative, you can (of course) choose any path you wish. If the above vector field was *not* conservative, then take the straight-line path from the origin to the final point. (Note; the vector field has been non-dimensionalized so your answer won't have units. That's unusual for us – but just roll with it for this problem.)

8. Based on the fact that $\int_{-\infty}^{\infty} f(x)\delta(x-a)\mathrm{d}x = f(a)$, what do the units of $\delta(x-a)$ have to be? (You may assume the unit of $x$ is meters).

9. Occasionally, we need to use a function called the "signum function" (often written $\mathrm{sgn}(x)$). This function is piecewise defined as:

$$\mathrm{sgn}(x) = \begin{cases} -1 & x < 0 \\ 1 & x > 0 \end{cases}$$

(in essence, it reports the sign of $x$). Write $\mathrm{sgn}(3x-1)$ in terms of $x$ and the $\delta$-function.