Assignment III, PHYS 409 (Electromagnetism I)
Fall 2019   Due 9/12/19 at start of class

Please supply your complete, legible, and well organized solutions on separate paper. No Mathematica, calculators, or other technological aids should be used (or are necessary) to complete this homework assignment.

1. A sphere with radius R is at the origin and carries a volume charge density $\rho(r) = \frac{k_1}{r}$.

   a) For ρ to be an actual volume charge density, what would the units of the unspecified constant k_1 have to be?

   b) Find the total net charge on the sphere.

   c) Find the magnitude of the resulting electric field as a function of r for $r < R$.

   d) Find the magnitude of the resulting electric field as a function of r for $r \geq R$.

   e) Sketch (by hand) a graph of $|\vec{E}|$ as a function of r.

   f) Show that your answer to part (d) reduces to the field from a point charge for $r > R$; the magnitude of the point charge should equal your answer to part (b).

2. Two concentric spheres have radii R_1 and R_2 ($R_1 < R_2$). The charge distribution in this alignment is as follows:

$$\rho(r) = \begin{cases} c_2 r & r < R_1 \\ -c_3 & R_1 < r < R_2 \\ 0 & r > R_2 \end{cases}$$

   a) Find the electric field everywhere. (This will be a piece-wise defined function over three regions).

   b) Find c_3 in terms of c_2, R_1, and R_2 if the total charge enclosed in a sphere of radius $r > R_2$ is 0.

   c) Use the relationship you developed in part (b) to determine what c_3 would be in terms of R if $R \equiv R_1$, and $R_2 = 2R$. (So, just to be clear, we're saying that there is zero total charge in the concentric sphere setup since the inner positive charge is exactly canceled by the outer shell of negative charge. We are also changing the notation a little by fixing $R_2 = 2R_1$ and calling those distances $2R$ and R respectively. Your task in this part of the problem is to find what c_3 must be in terms of c_2 and R.)

   d) Using the expression for c_3 developed in part (c), simplify your expressions for electric field you found in part (a) and report the electric field in the three regions (i) $r < R$, (ii) $R < r < 2R$, and (iii) $r > 2R$. Your answers should be in terms of R, r, c_2, and fundamental constants.

   e) By taking the line integral of $-\vec{E}$ found in part (d) from ∞ to 0 (which you'll have to do in three pieces), find the potential at the origin for the "zero-total-charge" configuration we've been working with in the last several parts of this problem. Note that even though this system has zero net charge, the potential at the origin is nonzero. (You may leave the numerical coefficient as the sum of two fractions if you wish). (It might not be a bad idea to make sure that your final answer has units of Volts).

3. An infinite sheet of charge exists in the $x-y$ plane with uniform surface charge density $\sigma(x,y)=\sigma_\circ$. From this infinite sheet of charge, a circular hole is cut of radius R. The circle's center is at $x=y=0$. Thus, you are left with an infinite sheet of charge, minus a disk at the origin.

   a) Before the hole was cut out, what was the electric field due to this sheet of charge at the position $H\hat{z}$ (with H some unspecified constant).

   b) Now that the hole has been removed, what is the electric field at the position $H\hat{z}$? (Hint: a hole is the same as the superposition of equal and opposite charges).

   c) If $H \gg R$, then the hole in the plane doesn't contribute much to the electric field at H. Show that your answer in part (b) reduces to your answer in part (a) when $H \gg R$.

   d) If $H \to 0$, show your answer in part (b) becomes what you'd expect. Explain.

4. An infinitely long cylinder has radius S and, inside the cylinder, there exists a charge density $\rho(s)=c_4 s^3$.

   a) If $\rho(s)$ is a volume charge density, what must be the units of c_4?

   b) What is the total charge per unit length enclosed in a cylinder concentric to the charged cylinder of radius s? (This is going to be a function of s).

   c) Find the electric field created by this charge distribution for $s<S$.

   d) Find the electric field created by this charge distribution for $s>S$.

   e) Make a sketch of the magnitude of the electric field as a function of s for this charge distribution. You should assume $c_4>0$.

   f) Find $V(s)$ in the domain $s<S$. [Note – you'll have to use the origin as your reference point.] (Additional question you should ask yourself but don't have to turn in – why can't we use ∞ as the reference point for this charge distribution?)

   g) Find $V(s)$ in the domain $s>S$ (again using the origin as the reference point).

   h) Explicitly verify your solution to part (f) above by taking the negative gradient of your answer and demonstrating this is equal to your answer to part (c).

   i) Explicitly verify your solution to part (g) above by taking the negative gradient of your answer and demonstrating this is equal to your answer to part (d).

   j) Hand draw a sketch of $V(s)$ for $s>0$.

5. Five charges, each with charge $-q$, are brought together to form the vertices of a tetrahedron. The vertices are at $\langle a,0,0\rangle$, $\langle -a,0,0\rangle$, $\langle 0,a,0\rangle$, $\langle 0,-a,0\rangle$, and $\langle 0,0,3a\rangle$. There is also a final (6th) charge of $+q$ at $\langle 0,0,-3a\rangle$. Take care – this final charge does have a different sign than the first five charges.

   a) Find the potential at position $h\hat{z}$. (h is an (unknown) constant). Assume infinity is your reference point.

   b) Calculate the total amount of work needed to assemble this charge arrangement. Please simplify your answer! I was able to get my answer down to two terms.

6. An infinite slab of charge exists parallel to the $x-y$ plane running from $z=-a$ to $z=a$. Inside this slab, there exists a volume charge distribution:

$$\rho = \begin{cases} c_5|z|^3 & |z| < a \\ 0 & |z| > a \end{cases}$$

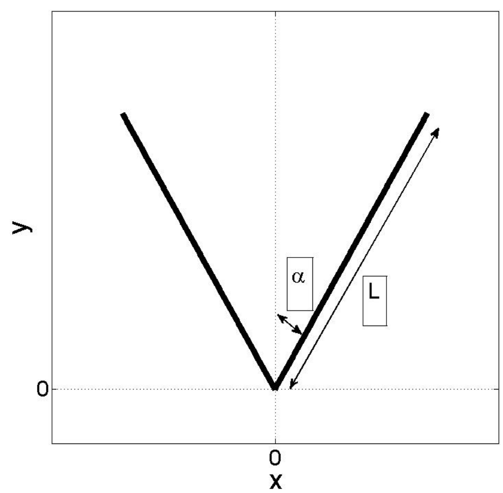
a) If ρ is a volume charge density, what must be the units of c_5?

b) What is the total charge enclosed in a box of dimensions $d \times d \times d$ centered at the origin? (This is going to be a function of d).

c) Find the electric field created by this charge distribution for $z < a$.

d) Find the electric field created by this charge distribution for $z > a$.

7. A wire of total length $2L$ is bent into a V shape as shown below. The half-angle of the V is α. The wire lies in the $x-y$ plane and the V is bisected by the y axis. (See picture for details). The line charge density on the wire is a constant $\lambda_\circ$.



a) What is the total net charge on the wire? (Don't overthink it).

b) Find the electric field created by this wire at the position $\vec{r} = h\hat{y}$ (h is some unspecified constant). (You will likely set up a reasonably ugly integral here. You may use any resource you wish to compute this integral including – but not limited to – *Mathematica*, MATLAB, Maple, Integral Tables, integrals.com, etc. If you enjoy messing with ugly integrals you may, of course, solve it by hand as well.)

c) A long distance from the wire ($h \gg L$), this charge distribution should "look" like a point charge. The magnitude of this point charge should be what you obtained in your answer to part (b) and the distance from the point charge should be h. Verify that your answer to part (c) for $h \gg L$ does, in fact, reduce to the field from a point charge of this magnitude and at this distance.