Assignment IV, PHYS 409 (Electromagnetism I)
Fall 2019 Due 9/17/19 at start of class

Please supply your complete, legible, and well organized solutions on separate paper. No Mathematica, calculators, or other technological aids should be used (or are necessary) to complete this homework assignment.

1. Let there be an infinite plane of charge of constant surface change density $\sigma_\circ$ in the $x-y$ plane (i.e. the plane is described by $z=0$). This is a problem where we can't take the zero for potential to be infinity because the electric field does not decay to zero at infinity.

   a) Find the electric field at some height z above the infinite plane of charge.

   b) Calculate $V(z)-V(z_1)$. (You may assume that $z>z_1>0$).

2. A sphere with radius R is at the origin and carries a volume charge density $\rho(r)=\frac{k_1}{r}$. (Yes, this is the same set-up as number 1 in the last HW assignment). Finding $V(r)$ everywhere for this one by integrating over $\rho/\text{𝓇}$ ends up being kind of an ugly integral. Instead, we'll find $V(r)$ by using $V(r)=-\int_*^r \vec{E}\cdot d\vec{\ell}$. We'll take $*$ as ∞ since we know the field decays to zero there. We calculated $\vec{E}(r)$ for this distribution in the last homework. You should have gotten (assuming I did it right myself):

$$|\vec{E}(r)| = \begin{cases} \frac{k_1}{2\epsilon_\circ} & r<R \\ \frac{k_1 R^2}{2\epsilon_\circ r^2} & r>R \end{cases}$$

   a) By taking the line integral of $\vec{E}$ from $*$ to r, find $V(r)$ for the range $r>R$.

   b) By taking the line integral of $\vec{E}$ from $*$ to r, find $V(r)$ for the range $r<R$. (Note that this will require you to break the integral up into two parts).

   c) Verify your solution by taking the negative gradient of $V(r)$ to find $|\vec{E}|$ in the range $r<R$

   d) Verify your solution by taking the negative gradient of $V(r)$ to find $|\vec{E}|$ in the range $r>R$.

   e) Calculate the total amount of work needed to assemble this sphere. (If you get ∞, it is probably because you are using equation 2.45. That's not the right answer. To understand why you get that, read page 95. For this problem, we're going to assume the charge already exists and we're moving it from ∞ to the sphere. Therefore, equation 2.45 isn't the way to go.)

   f) Explicitly verify that your answer to part (e) has the correct units.

3. As you've probably noticed by now, I typically break apart most problems into small steps for you to help guide you to an answer. Every now and then, however, I want to make sure you understand the big picture so I'll give you a problem setup and just ask you to go all the way to the answer in one big step; this is one of those problems.

   Coaxial cylinders of radii a and b (with $a < b$) run along the z axis. There is no volume charge density anywhere, but on the *surface* of both cylinders there *is* a constant surface charge density. The net charge of the entire configuration is 0 (so the signs of the surface charge density at radius a and b are opposite each other, and the magnitude of the charge density on the two surfaces is *not the same*). For sake of having standard notation, we will call the total charge on the surface of the radius a cylinder for each unit-length along the z axis λ. (Just to be clear, this means that if a segment of length ℓ is cut from the cylinder of radius a, a total surface charge of $\lambda\ell$ would be uniformly smeared over its surface). Find the total energy per unit length required to assemble this charge configuration in terms of λ, a, b, and other fundamental constants of nature.