Assignment VI, PHYS 409 (Electromagnetism I)
Fall 2019   Due Tuesday, October 8th, 2019 at beginning of class

Please supply your complete, legible, and well organized solutions on separate paper. No Mathematica, calculators, or other technological aids should be used (or are necessary) to complete this homework assignment.

I know you have been getting a lot of homework in this class lately, but this is the last “normal” homework before your second midterm, so I wanted to make sure you had a chance to do a couple practice problems like you might expect on your next exam. I promise this is as short as I could manage, without throwing you into the fire of an exam without sufficient practice.

1. Write $3\cos(3\theta)+2\sin^2(\theta)-5\cos(\theta)+3$ as a sum of Legendre Polynomials with constant coefficients.
2. (Here's another one where I don't break it into parts for you but just set you loose.) A spherical shell of radius a has potential on its surface $V_\circ(\cos(4\theta)+2)$. (I know – tantalizingly close to what we did in class!) Assume all of the charge that generated this potential exists on the surface of the sphere (there are no charges anywhere else inside the sphere or outside of the sphere). Find the potential everywhere (inside and outside of the sphere). Also, find the surface charge density that brings about this potential.
3. Charge $-7q$ is located at $\langle 0,0,2a\rangle$, charge $+2q$ is located at $\langle a,0,0\rangle$ and another charge $+2q$ is located at $\langle -a,0,0\rangle$. Expand the potential from these three charges in a multipole expansion and write down the first two nonvanishing terms. (In spherical coordinates $V(r,\theta)$.)
4. A sphere of radius R carries charge density $\rho(\vec{r}) = kr^2\cos\theta$.
   a) Find the total charge inside this sphere.
   b) Find the dipole moment of this sphere.
   c) Based on your answers to parts (a) and (b), find the approximate potential a long distance from this sphere. (In spherical coordinates, $V(r,\theta)$). (You only need to keep the monopole and dipole terms).
5. Your book states that the dipole moment of a charge distribution is only independent of origin if the net charge of the system is zero. We're going to help verify this by looking at the dipole moment of four different systems. The first two are true quadrupoles. The last two are not quite quadrupoles. For each of the four parts below, I want you to actually calculate the quantity expected even if you think you know it based on symmetry or other properties.
   a) Find the dipole moment of the following set of four charges: Charge 1 = charge $-q$ at the origin. Charge 2 = charge $+q$ at $\langle 0,0,2d\rangle$. Charge 3 = charge $+q$ at $\langle 0,2d,0\rangle$. Charge 4 = charge $-q$ at $\langle 0,2d,2d\rangle$.
   b) Find the dipole moment of the following set of four charges: Charge 1 = charge $-q$ at $\langle 0,-d,-d\rangle$. Charge 2 = charge $+q$ at $\langle 0,-d,d\rangle$. Charge 3 = charge $+q$ at $\langle 0,d,-d\rangle$. Charge 4 = charge $-q$ at $\langle 0,d,d\rangle$.
   c) Find the dipole moment of the following set of four charges: Charge 1 = charge $-2q$ at the origin. Charge 2 = charge $+q$ at $\langle 0,0,2d\rangle$. Charge 3 = charge $+q$ at $\langle 0,2d,0\rangle$. Charge 4 = charge $-q$ at $\langle 0,2d,2d\rangle$.
   d) Find the dipole moment of the following set of four charges: Charge 1 = charge $-2q$ at $\langle 0,-d,-d\rangle$. Charge 2 = charge $+q$ at $\langle 0,-d,d\rangle$. Charge 3 = charge $+q$ at $\langle 0,d,-d\rangle$. Charge 4 = charge $-q$ at $\langle 0,d,d\rangle$.