Assignment VIII, PHYS 409 (Electromagnetism I)
Fall 2019 Due Tuesday, October 29, 2019

1. The bound charge volume and surface densities ρ_b and σ_b typically arise due to the polarization of an initially uncharged dielectric. Therefore, though charge moves around a little bit, the net charge on the dielectric should remain zero. From the definitions of ρ_b and σ_b, mathematically demonstrate that the total enclosed bound charge in a neutral dielectric must be zero.

2. Two infinite conducting cylindrical shells have radii a and b $(a < b)$. The outer cylinder carries total charge $-\lambda$ per unit length, while the inner cylinder carries total charge $+2\lambda$ per unit length. The space between the cylinders contains a dielectric with constant permittivity ϵ $(\epsilon > \epsilon_\circ)$. Find the Displacement field everywhere and the Electric field everywhere.

3. Two concentric conducting spherical shells, with radii a and $3a$, have charge $+Q$ and $-Q$ respectively. The space between the shells if filled with a linear dielectric with permittivity:

$$\epsilon(r) = \frac{3\epsilon_\circ a}{4a - r}$$

   which varies with radial distance r from $\epsilon_\circ$ at $r = a$ to $3\epsilon_\circ$ at $r = 3a$.

   a) Find the displacement field everywhere.
   b) Determine the bound charge density between the spherical shells.

4. A sphere of linear dielectric material has embedded in it a free charge density of the form $\rho_{\text{free}}(r) = \xi r^2$ (for $r < R$). Find the potential at the center of the sphere (relative to infinity) if its radius is R and the dielectric constant is κ (e.g. $\epsilon = \kappa\epsilon_\circ$).

5. A dielectric sphere of radius a and dielectric constant κ is placed between the plates of a very large parallel plate capacitor, so that the field everywhere inside the sphere (if the dielectric weren't there) would be uniform: $\vec{E} = E_\circ\hat{z}$. Find the electric field actually inside the dielectric sphere. Leave your answer in terms of $E_\circ$, κ, a, and/or any fundamental constants. *Hint: if* $\kappa = 1$, *your answer should be* $\vec{E} = E_\circ\hat{z}$*!*