Assignment IX, PHYS 409 (Electromagnetism I)
Fall 2019   Due Tuesday, November 5, 2019

1. An air-filled parallel plate capacitor is charged with a source that produces a voltage difference between the plates $V_\circ$. The capacitor is then removed from the voltage source. The capacitor is then placed in a (linear, isotropic) dielectric fluid.

    a) If the voltage difference between the plates of the capacitor is now $\alpha V_\circ$, what is the dielectric constant of the fluid?

    b) Would you expect α to be greater than or less than 1? Explain.

2. Let Medium 1 (existing everywhere that $z < d$) be a vacuum (or air, if you prefer) and medium 2 (existing everywhere that $z > d$) be a linear, isotropic dielectric with dielectric constant κ. If the electric field in medium 1 can be written:

$$\vec{E} = E_\circ \left(\hat{x} + 2\hat{y} + 3\hat{z}\right) \qquad (z < d)$$

   and we know there is no free charge on the interface between the media, what is $\vec{D}$ in medium 2?

3. Are lines of $\vec{E}$ bent toward or away from the normal to the interface when going into a dielectric in which κ is larger? (If you need to, assume there is no free surface charge at the interface).

4. A sphere of radius R has a uniform dielectric constant κ. A charge $+q$ is at the center of the sphere. Find the polarization field $\vec{P}$ in the sphere and the total bound charge on the surface of the sphere.

5. This is going to be very similar to a problem we did in class. Consider a parallel plate capacitor with extremely large surface area and a separation between the plates of $5a$. In between the plates of the capacitor, there are three dielectrics sandwiched between. If we let the bottom plate of the capacitor be at $z = 0$, we fill the medium between $z = 0$ and $z = 5a$ as follows:

$$\epsilon_r = \begin{cases} 4 & 0 \le z \le 2a \\ 2 & 2a \le z \le 4a \\ 1 & 4a \le z \le 5a \end{cases}$$

   In other words, we have three different layers inside the capacitor – with the top-most layer being a vacuum).

    a) Find $\vec{D}$ in each of the three layers between the plates

    b) Find $\vec{E}$ in each of the three layers between the plates

    c) Find $\vec{P}$ in each of the three layers between the plates

    d) Find $|\Delta V|$ between the plates, assuming a charge density of σ on the top plate and $-\sigma$ on the bottom plate.

    e) Find the bound charges at each of the 6 interfaces.

6. Let there be an infinitely long hollow cylindrical tube of inner radius a and outer radius b. Further, let the volume current density in the tube be given by:

$$\vec{J} = \begin{cases} \left(\frac{k_1}{s^4} + k_2 s^3\right)\hat{z} & a < s < b \\ 0 & \text{otherwise} \end{cases}$$

   a) What are the units of k_1?
   b) What are the units of k_2?
   c) What is the total current running through this tube?

7. Let there be a sphere of radius R and total charge Q. Let the volume charge density be given by:

$$\rho(r) = \begin{cases} \frac{36Qr^3}{20\pi R^6} & 0 < r < R \\ 0 & \text{otherwise} \end{cases}$$

   You should find that if you integrate $\iiint \rho \, d\tau$, you don't obtain Q. The reason for this is that – in addition to the volume charge density – there is some extra (negative) surface charge on the outer surface of the sphere. This surface charge is evenly distributed on the surface of the sphere so that the total charge (from the inner volume of the sphere and its surface) add up to exactly Q.

   a) Find the surface charge density.
   b) Let the sphere rotate at a constant angular velocity ω. Find the volume current density $\vec{J}$ as a function of r, θ, and ϕ.
   c) Explicitly verify your answer to part (b) has the proper units for a volume current density.
   d) Let the sphere rotate at a constant angular velocity ω. Find the surface current density $\vec{K}$ as a function of r, θ, and ϕ.
   e) Explicitly verify your answer to part (d) has the proper units for a surface current density.