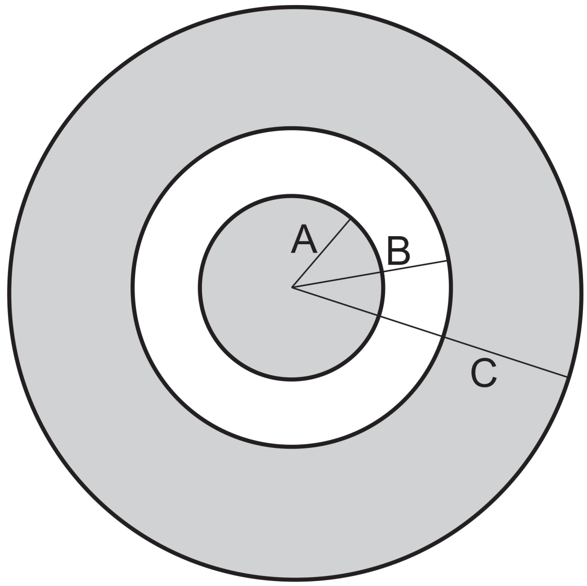
Assignment XI, PHYS 409 (Electromagnetism I)
LAST ONE (WooHoo!)
Fall 2019 Due Monday, December 2nd by 5 PM

Either email me a PDF of your solutions to LarsenML@cofc.edu by 5 PM on December 2nd or drop them off in my office (RITA 317) between noon and 5 PM on Monday, December 2nd. No late work will be accepted.

1. Current is distributed on the interior of a wire of infinite length and radius R through the relationship $J = ks^2\hat{z}$ where $\hat{z}$ is the direction along the wire's axis. The wire is made of a linear magnetic material with susceptibility χ_m.
   a) What is $\vec{B}(s)$ everywhere?
   b) What is $\vec{H}(s)$ everywhere?
   c) What is $\vec{M}(s)$ everywhere?
   d) Take the curl of M to find the bound current volume density $\vec{J}_b$.
   e) Find the bound surface current density $\vec{K}_b$.
   f) Integrate your answers to part (d) and (e) (over the appropriate volumes or surfaces) to find the net bound current flowing down the wire. (Briefly comment on your answer).
2. A fat wire, having radius a, carries a constant current I that is uniformly distributed over its cross sectional area. A narrow gap in the wire, of width $w \ll a$, forms a parallel plate capacitor. The gap is filled with an insulating material with permittivity ϵ and permeability μ. Find the magnetic field $\vec{B}(s)$ in the gap, with distance s from the central axis of the wire small compared to a, so that edge effects can be ignored.
3. A square loop of wire, side length a, lies midway between 2 long wires that are $5a$ apart. Far away, the long wires are connected at both ends to form a loop. The loops are in the same plane, and 2 opposite sides of the small square loop are parallel to the long wires. A clockwise current is gradually increasing at a constant rate (i.e. $\frac{dI}{dt} = k$) in the small square loop. Find the emf induced in the big loop and determine which way the current will flow.

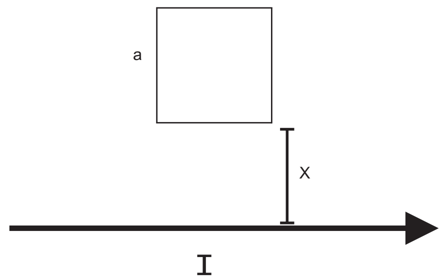


4. This problem is pretty important for practical applications. Let there be a coaxial cable. (This is a cable with two very long cylindrical tubes). Let the two tubes be separated by an insulating material of magnetic susceptibility χ_m. Let a current I flow through the inner conductor (out of the paper) and return along the outer conductor (flowing into the paper). In both cases, the current distributes itself uniformly over the *surface* of the conductor.

   a) Calculate the magnetic field in the region between the tubes.

   b) Calculate the Magnetization between the tubes.

   c) Calculate the surface and volume bound currents (if any) in the region between the tubes.



5. Yup – we get to play with this system again. Like HW 10, we have an infinite wire carrying current I from left to right a distance x away from the closest part of a wire loop in a square shape with side-length a. Unlike last time, however, there is no current in the wire loop in the steady-state condition where I is constant in the infinite wire.

   a) Let the current in the infinite wire be described as follows. (τ is some positive constant).

   $$I(t) = \begin{cases} I_\circ & t < 0 \\ I_\circ e^{-t/\tau} & t > 0 \end{cases}$$

      i) Find an expression for the emf in the wire loop as a function of time.
      ii) Let the loop have total resistance R. What is the current (including an unambiguous assignment of direction!) as a function of time.
      iii) Assuming the loop still has total resistance R, add up all of the charge moving past any point in the loop for all time.
      iv) Find the total energy dissipated through Joule heating in this scenario.

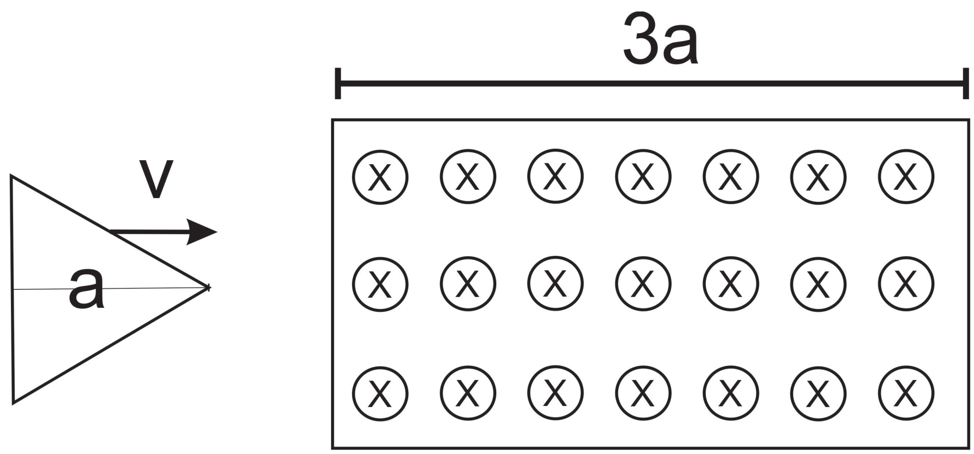
   b) Let the current in the infinite wire be described as follows. (ω is some positive constant).

   $$I(t) = \begin{cases} I_\circ & t < 0 \\ I_\circ \cos(\omega t) & t > 0 \end{cases}$$

      i) Find an expression for the emf in the wire loop as a function of time.
      ii) Let the loop have total resistance R. What is the current as a function of time. Qualitatively describe the behavior.
      iii) Assuming the loop still has total resistance R, find the *net* total charge that has passed a given point in the loop *as a function of time.*

   c) Let the wire carry constant current $I_\circ$. The loop is pulled with uniform speed v directly away from the wire (i.e. up the page).

      i) Find the emf induced in the loop as a function of time.
      ii) Find the direction of the induced current, assuming $v > 0$.



6. An equilateral triangular wire with bisector length a (as shown) and resistance R moves from left to right at uniform velocity v. At time t_1, the leading tip of the triangle starts entering a region of uniform magnetic field $\vec{B}$ into the page. The magnetic field extends for a distance $3a$ along the direction of the wire's movement as shown.

   a) Find $|\mathscr{E}(t)|$ in the wire.

   b) Write an expression for the total net charge that has moved past a given point in the triangular loop between t_1 and $t = \infty$.