Assignment IV, PHYS 459 (Cloud and Precipitation Physics)
Fall 2019 Due Tuesday, September 17th, 2019 at Beginning of Class

1. In class, we began to talk about how to best represent the relative abundance of different sized aerosol particles by plotting what is known as a size-distribution. Plotting size distributions is extremely common in aerosol, cloud, and precipitation physics – but the conventions regarding how to display the information and the units used can vary from one sub-field to another. Further, it takes quite a bit of practice to get used to understanding $y-$axes with labels like $\frac{\mathrm{d}N}{\mathrm{d}(\log D_p)}$ $\left[\mathrm{cm}^{-3}\mu\mathrm{m}^{-1}\right]$. Rather than head down this road, I hope we can develop *some* sense of the various ways that the distribution of sizes can manifest themselves depending on context with this problem.

   On the course webpage, I have made available for download a sample data-file acquired by Chris Blouin using an optical particle counter in my lab. (Download the data from `http://larsenml.people.cofc.edu/phys459_hw04_data.txt`.) This device (the CLiMET CI-8060) draws in air at 1 cubic foot per minute (28.3168 L/min) through a narrow tube. As the air in the tube moves through the instrument, the air flow is illuminated by a bright white light source. A photo-diode then continuously measures the strength of the scattered light. When an aerosol particle is illuminated, the photo-diode measures a voltage which is then converted into an approximate particle diameter through a formula that I am approximating for you. IN general, the detector is sensitive to aerosol particles between about 300 nm and 10 μm in diameter; particles smaller than 300 nm can not be detected with this instrument.

   The first column of the supplied file indicates the detection times of individual aerosol particles (in seconds since the measurement started). The second column of the supplied file indicates the associated estimated diameter of the aerosol particle (in micrometers). This data-set is far from perfect, but hopefully will serve as a useful teaching tool to get a sense of real parameters.

   In the questions that follow, assume the data given to you is error-free (e.g. we will remove any consideration of sizing or detection errors); all detected aerosol particles will be assumed to be perfectly spherical with the diameters stated, and made out of something with the same density of liquid water (1000 kg/m^3).

   For this entire problem, you may use any computational resources you wish. Excel, MATLAB, and Mathematica are all available in the computer labs within the department.

   a) What is the mean detected particle diameter for the data provided?

   b) What is the standard deviation of the detected particle diameters for the data provided?

   c) The sample is drawn in through a sampling sub-volume of diameter 0.25 inches. A total volume of approximately 28.3168 liters per minute is drawn through this sampling sub-volume. Using the diameter of the tube as the characteristic length-scale of the problem, what is the approximate Reynolds number of the air flow through this instrument? Assume air at near-surface density and at 20 Celcius. (Note – people who have had fluids – don't assume a parabolic profile for air flow. Just assume it is uniform across the 0.25 inch diameter cross-section).

   d) Assuming that the entire 28.3168 liters per minute drawn through the instrument is "seen" by the photodiode, what would the total particle loading (in detected particles/cubic centimeter) be for this data-set?

   e) What is the average surface area (μ_A) of the detected particles? (This is different than calculating the average radius, squaring it, and multiplying it by 4π.)

f) Calculate the area-weighted mean diameter of the detected particles by computing $\left(\frac{\mu_A}{\pi}\right)^{1/2}$.

g) What is the average volume (μ_V) of the detected particles?

h) Calculate the volume-weighted mean diameter of the detected particles by computing $\left(\frac{6\mu_V}{\pi}\right)^{1/3}$.

i) You should have found that the volume-weighted mean diameter of the detected particles is larger than the mean diameter of the particles. Must this inevitably be the case? Explain why or why not. [Hint – look up Jensen's inequality.]

j) Atmospheric particulates (aerosol particles) can have impacts to human health. The way that this risk is assessed is to calculate a quantity known as $PM_{2.5}$, which measures the total mass concentration of particulate matter with diameters less than 2.5 μm. The US EPA standard for maximum safe exposure is 35.0 $\mu g/m^3$ (24-hour average) or 27.0 $\mu g/m^3$ (annual average). Calculate the $PM_{2.5}$ reading for my lab and determine if the air quality is dangerous based on the data set supplied.

k) As noted above, no particles with diameters less than 300 nm are detected with this optical particle counter. That being said, I'm reasonably confident that including sub 300nm particles wouldn't affect our answer to part (j) very much. Why not?

2. In class, Dr. Larsen did (or will) show that Stokes' drag is appropriate for a 100 nm diameter aerosol particle falling at its terminal velocity through air, but it is not appropriate for a 1 mm diameter raindrop falling at its terminal velocity through air. Assume the $Re \ll 1$ condition means that, strictly speaking, you need $Re < 0.1$ and determine what is the maximum drop diameter appropriate for Stokes' drag in the atmosphere. (You may assume the particle is spherical and near the surface of the earth, it is falling through still air at 5°C, the density of the particle is 1000 kg/m^3, and the density of air is 1.25 kg/m^3.