Assignment VI, PHYS 459 (Cloud and Precipitation Physics)
Fall 2019 Due Thursday, October 10th, 2019 at beginning of class

Please supply your complete, legible, and well organized solutions on separate paper.

1. In class, when working on homogeneous nucleation, we developed the following expression for ΔG:

$$\Delta G = 4\pi r^2 \sigma - \frac{4}{3}\pi r^3 nkT \ln\left(\frac{e}{e_s}\right)$$

   a) Find the value of r when ΔG reaches a maximum. (We call this value r_c for the critical radius). You may assume all parameters in the problem except for r can be treated as constants. You may also assume $e > e_s$.
   b) Develop a (fully simplified) expression for ΔG when $r = r_c$.
   c) For $e > e_s$, $\Delta G = 0$ at two values of r. The first is at $r = 0$. Find an expression for the other one. We'll refer to the value of r you find here as $r_{\text{2nd zero}}$ (in the following parts of this problem).
   d) Let's compare $r_{\text{2nd zero}}$ to r_c. You might expect $r_{\text{2nd zero}} = 2r_c$ if the curve is completely symmetric for positive values of ΔG. Is $r_{\text{2nd zero}}$ greater than, equal to, or less than $2r_c$?
   e) How many liquid water molecules would be in a sphere of radius $r_{\text{2nd zero}}$? Assume $e/e_s = 1.042$, $\sigma = 0.076$ J/m^2, and $T = 270$K.

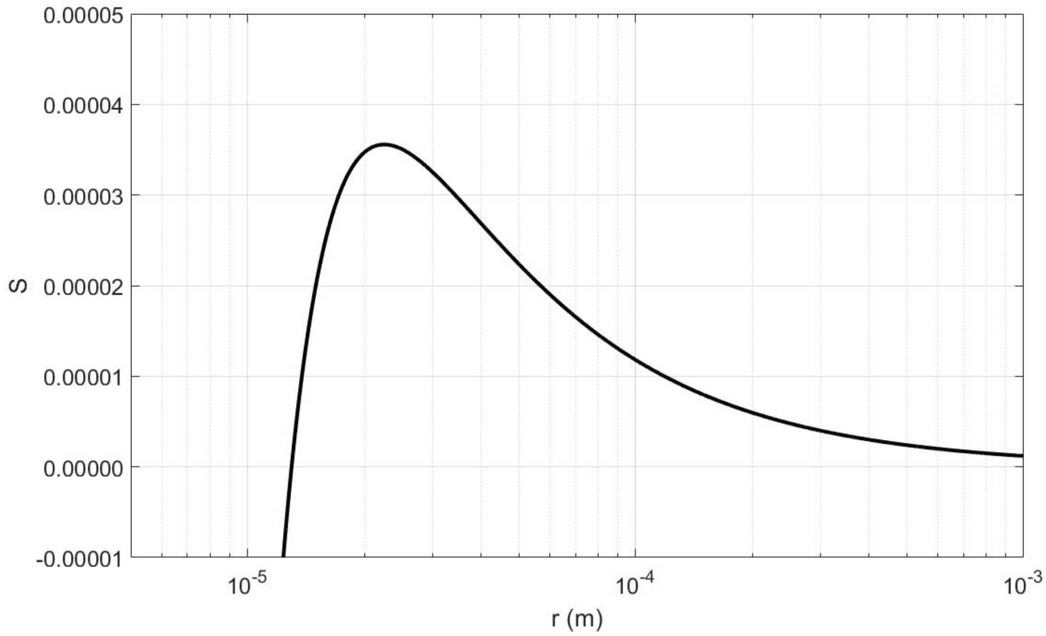
2. In Class, Dr. Larsen argued that – for homogeneous nucleation – the critical radius is given by:

$$r_c = \frac{2\sigma}{nkT \ln(e/e_s)}$$

   Use a computer program of some sort to generate a publication-quality figure of r_c as a function of e/e_s – ranging from 1.00001 to 1.02. (S = .001% to 2%). You may assume $T = 270K$ and, despite the temperature, all water involved is either in the liquid or vapor phases. I don't need your code for this; just give me a printout of the figure. Make your y-axis logarithmic.

3. For the case of an insoluble spherical cloud condensation nucleus, the basic equation for nucleation is structurally the same as Kelvin's equation (used for homogeneneous nucleation). However, to calculate the total number of water molecules you need to reach the associated (unstable) equilibrium, you can subtract the volume of the aerosol particle. (Think of a peach – the aerosol is like the pit and the fruit part you eat is like the collection of water molecules). Calculate the total number of liquid water molecules necessary to reach the critical radius in a (very high) ambient supersaturation of 0.01. Assume the temperature is 275K and the base aerosol particle has a radius of 38 nm.

4. Examine the sample Köhler curve below and answer the following questions based on it.



a) If a droplet with 10 μm radius is exposed to a supersaturation of 2×10^{-5}, what would its equilibrium radius be?

b) If a droplet with 40 μm radius is exposed to a supersaturation of 2×10^{-5}, what would its equilibrium radius be?

c) If a droplet with 200 μm radius is exposed to a supersaturation of 2×10^{-5}, what would its equilibrium radius be?

d) We state that an equilibrium is stable if a small perturbation in either direction results in a tendency to cancel the perturbation. Given that definition, are there any points on the Köhler curve that are stable? If so, explain and state where they are on the plot above. If no points like that exist, explain why not.

e) Given that I used $a = 1.2 \times 10^{-9}$ m, what must I have used for b to create this curve?

5. If we write $S = \frac{a}{r} - \frac{b}{r^3}$, what is the critical supersaturation in terms of a and b?

6. Given what you've learned about nucleation theory, write a succinct (1-3 sentence) reply to the following statement: "When relative humidity is at 100%, it means that there is a dynamic equilibrium between liquid and vapor states, so that liquid-vapor interfaces don't have a net tendency to favor a general movement from one phase to the other."