Assignment VII, PHYS 459 (Cloud and Precipitation Physics)
Fall 2019 Due Thursday, October 24th, 2019 at beginning of class

Please supply your complete, legible, and well organized solutions on separate paper.

1. Assume that the equation governing condensational growth of a water droplet (after activation) follows an equation of the following form:
$$r\frac{\mathrm{d}r}{\mathrm{d}t} = \frac{S}{A+B}$$
with A and B taking the place of really complicated constants involving pressures, temperatures, latent heats, coefficients of thermal conductivity, diffusion constants, and a whole other world of ugliness. For now, just let A and B be generic constants. S is the supersaturation. r is the radius of the particle. Assume you have a freshly minted water droplet with radius 100 nm that, you find, groww via condensational into a water droplet with radius 1 micron in about a second. (This is realistic, trust me). How long would it take this drop to grow via condensation to the size of a typical raindrop? (Radius around 600 micrometers). (Don't assume a value for G – use the observation that it went from 100 nm to 1 micron in 1.0 seconds to answer the question!)

2. A better (more realistic, but much harder to solve for r) equation that describes the rate of condensational droplet growth can be written:
$$r\frac{\mathrm{d}r}{\mathrm{d}t} = G\left(S - \frac{a}{r} + \frac{b}{r^3}\right)$$
does the equation used in the previous problem (a.k.a. $r\frac{\mathrm{d}r}{\mathrm{d}t} = \frac{S}{A+B} = GS$) over- or under-estimate the drop growth rate for a water droplet for $S > S_{\mathrm{crit}}$ and $r \approx r_c$? Justify your answer with a coherent explanation.

3. In class, I argued that the condensation process tends to cause a distribution of droplet sizes to become less dispersed (in other words, condensation narrows the size distribution). This is because small droplets grow faster than large droplets, and ultimately allows the small drops to partially "catch up" to the larger ones. At some point, we will talk about gravitational coagulation, which follows this basic form:
$$\frac{\partial m}{\partial t} \propto \begin{cases} r^6 & \text{for } 10 \leq r \leq 50\ \mu\text{m} \\ r^3 & \text{for } r \gtrsim 50\ \mu\text{m} \end{cases}$$
Based on this, figure out if gravitational coagulation should cause a size distribution to become narrower or wider. Assume the density of water is constant, and the mass of a drop is equal to $\rho_{\mathrm{water}}V$.

MORE ON BACK!

4. The saturation vapor pressure over liquid water for air at -20 Celcius is right about 1.280 mbar. At the same temperature, the saturation vapor pressure over ice is about 1.033 mbar.

   a) We'll use $G = 7.7 \times 10^{-11}$ m^2/s in the condensational growth equation. How long would it take a supercooled liquid water drop to grow from 20 micron *diameter* to twice its starting *VOLUME* if the environmental vapor pressure is 1.281 mbar?

   b) Let's use the same value of G for ice. (It actually isn't the same – but it is reasonably close-ish, and we'll keep it the same to keep the analysis simple). How long would it take a "spherical" ice crystal to grow from 20 micron *diameter* to twice its starting *VOLUME* if the environmental vapor pressure is 1.281 mbar?

   c) How long (in days) would it take for the 20 micron diameter supercooled water droplet to grow to the size of a raindrop (diameter 1 mm) under the same conditions as in part (a) above?

   d) How long (in hours) would it take for the 20 micron diameter spherical ice crystal to grow to the size of a raindrop (diameter 1 mm) under the same conditions as in part (b) above?

5. The largest difference between the saturation vapor pressure of liquid water and the saturation vapor pressure of ice occurs near about -14C. Thus, this is the regime where you would expect supersaturations with respect to ice to be largest and you would get most rapid growth of ice crystals by deposition.

   a) If a mixed phase cloud is exposed to a situation where the air is at -14C and supersaturated with respect to ice but subsaturated with respect to supercooled liquid water (as in the Wegener-Bergeron-Findeisen process), what sort of ice crystal habit would you expect to find a lot of? (Hint – you might have to google an image I showed in class).

   b) If a mixed phase cloud is exposed to a situation where the air is at -14C and supersaturated with respect to both ice and supercooled liquid water, what sort of ice crystal habit would you expect to find a lot of?

   c) What ice crystal habit would you expect in cirrus clouds that somehow have lots of water vapor? (Look up properties of cirrus clouds if you have to).