Assignment II, PHYS 459 (Cloud and Precipitation Physics)
Fall 2021 Due Thursday, September 2nd, 2021 at Beginning of Class

1. How many molecules are in the Earth's atmosphere? (Assume a spherical Earth. You may look up the radius of the Earth and assume the atmosphere is entirely Nitrogen gas N_2.)

2. How high would you have to go above sea level to observe a number concentration of approximately one molecule per cubic meter? (You may assume an isothermal atmosphere).

3. If the atmosphere was incompressible with constant density of 1.25 $\mathrm{kg/m^3}$ how thick would it be?

4. As noted in class, pressure can be written as a function of height via:

$$P(z) = P_\circ \exp(-z/H)$$

where we will use $H = 8500$ m. If you compare the atmospheric pressure in Denver to the atmospheric pressure in Charleston, Denver will *always* be much lower. However, for meteorological purposes, we would often like to take the elevation dependence out of the picture. (Why might we want to do this? Let's say that City A normally has an atmospheric pressure (due to its elevation) of 700 mmHg while City B normally has a pressure of 750 mmHg. On a particular day, barometers in both cities might read 730mmHg. That is higher than usual for city A while lower than usual for city B, despite being the same pressure. If we want to be able to compare "apples to apples" when using weather maps, we need to put everyone on the same scale. As such, it is very normal to take a barometer reading (called "station pressure") which measures P at a particular latitude, longitude, and altitude and convert it to sea-level equivalent pressure ($P_\circ$(latitude, longitude, sea-level). If we didn't do this, weather maps would always show a low-pressure region over the Rockies while the coasts would be perpetually high pressure regions, and they'd be pretty useless for actually forecasting the weather. Weather maps nearly always use sea-level equivalent pressure instead of station pressure. As I write this, the airport gives the current atmospheric pressure in Charleston as 30.10 inches of mercury (abbreviated as inHg). You can treat our elevation to be essentially sea-level, so the pressure here right now is a little bit higher than the "standard" 29.92 inHg.

   a) If we were to take the same sea-level equivalent pressure in Denver (elevation 5225 ft.) that we have as I write this in Charleston (30.10 inches of Mercury), what would a barometer in Denver actually read? (You'll have to convert feet to meters). Give me the equivalent Denver station pressure in units of mmHg.

   b) Do the same thing, except tell me what a barometer would read at the top of Mount Everest (8848 meters), and give your results in atmospheres.

   c) Do the same thing, except tell me what a barometer would read in death valley (elevation 86 m BELOW sea level), and give your results in mm Hg.

   d) If a barometer reads 729 mmHg in Green Bay, WI (elevation 177 m), what would a barometer (that measures in mmHg) read in Kearney, NE (elevation 656 meters) if both cities have the same sea-level equivalent pressure?

MORE ON BACK!

5. An approximate fit to the Clausius-Clapeyron equation that gives saturation vapor pressure (in mbar) as a function of temperature (in Celcius) is given by the following equation:

$$e_s(T) = 6.112 \exp\left(\frac{17.67T}{T + 243.5}\right)$$

   Use the above equation (and potentially a calculator or MATLAB or another tool to evaluate the expression) to determine the following:

   a) If the external temperature is 83 degrees Fahrenheit and the current water vapor pressure is 10 hPa, what is the approximate Relative Humidity?
   b) If the external temperature is 45 degrees Fahreneheit and the current water vapor pressure is 10 hPa, what is the approximate Relative Humidity?
   c) If the external temperature is 83 degrees Fahrenheit and the current water vapor pressure is 10 hPa, what is the approximate dew point temperature (in Fahrenheit)?
   d) If the external temperature is 45 degrees Fahrenheit and the current water vapor pressure is 10 hPa, what is the approximate dew point temperature (in Fahrenheit)?

6. The absolute humidity is known to be 4.0 g/m^3 and the weatherperson on the news says it is 72 degrees (F) outside. You (and the TV station/weather center) are both at an altitude of 1200 meters above sea level, and you may assume that the density of dry air decreases with the hydrostatic relation and has a value of 1.25 kg/m^3 at sea level. So all of our answers are the same, use 8500 meters as the scale height of the atmosphere. Clearly state any assumptions you have to make. What is:

   a) The specific humidity?
   b) The mixing ratio?
   c) The relative humidity?

7. An impermeable membrane near the surface of the earth divides a rigid, well-insulated tank into two equal sub-volumes. The left side contains air at 40 degrees Fahrenheit at 60% relative humidity. The right side contains air at 75 degrees Fahrenheit at 95% relative humidity.

   a) What is the current water vapor pressure on the left hand side?
   b) What is the current water vapor pressure on the right hand side?
   c) If the membrane is punctured and the two volumes of air are allowed to mix, the temperature of the combined gas will be the arithmetic average of the two temperatures. The total amount of water vapor in the box is not going to change. What is the relative humidity of the mixed gas? (Assume that the density of air on both sides of the membrane before it is punctured is the same.)

8. It may seem strange to you that we asserted that atmospheric pressure at some height z depends only on the total weight of the air above height z. After all, the air above you is not really in direct contact with you, so how can a Nitrogen molecule 30 km above your head have direct influence on the pressure you feel? To help this make more sense, we'll look at the force associated with an ordinary mechanical collision (think back to PHYS 111)....

   Consider a perfectly elastic ball of mass m bouncing up and down on a horizontal surface under the influence of a constant downward gravitational acceleration g. The ball is initially dropped from height h and (since the ball is perfectly elastic) always rebounds to this same height. Calculate the average force applied to the horizontal surface by the ball.

   Hints/clues/observations: Most of the time – while the ball is in the air – there is no force applied on the surface by the ball. During the bounce, however, the ball will impart momentum to the surface. After n bounces, this momentum will have been applied to the surface n times. Recall the definition of a force is $\vec{F} = \frac{\Delta\vec{p}}{\Delta t}$ where $\Delta\vec{p}$ is the momentum change of the ball. Since I only gave you parameters m, g, and h in the problem statement, it is reasonable to assume your final answer will only depend on these variables (or a subset of them) along with (possibly) fundamental constants of nature.

   The math here is simple, so please provide enough words so that I can understand what you are doing.