Assignment V, PHYS 459 (Cloud and Precipitation Physics)
Fall 2021 Due Thursday, September 23rd, 2021 at beginning of class

NOTE: If all goes according to plan, this will be your last homework assignment before the first midterm exam.

1. In class, Dr. Larsen showed that standard Stokes' drag is appropriate for a 1 μm diameter aerosol particle falling at its terminal velocity through air, but it is not appropriate for a 1 mm diameter raindrop falling at its terminal velocity through air. Assume the $Re \ll 1$ condition means that, strictly speaking, you need $Re < 0.1$ and determine what is the maximum drop diameter appropriate for Stokes' drag in the atmosphere. (You may assume the particle is spherical and near the surface of the earth, it is falling through still air at 5°C, the density of the particle is 1000 kg/m^3, and the density of air is 1.25 kg/m^3.

2. In class, we went into some detail to calculate the terminal velocity of an aerosol particle falling in still air. Eventually, we developed the equation:

$$|\vec{F}_{\text{drag}}| = \frac{\rho_f \pi D_p^2 C_D v_t^2}{8 C_C}$$

with:

$$C_C = 1 + \mathrm{Kn}\left(1.257 + 0.4e^{-1.1/\mathrm{Kn}}\right)$$

$$C_D = \begin{cases} \frac{24}{\mathrm{Re}} & \mathrm{Re} < 0.1 \\ \frac{24}{\mathrm{Re}}\left(1 + \frac{3}{16}\mathrm{Re} + \frac{9}{160}\mathrm{Re}^2 \ln(2\mathrm{Re})\right) & 0.1 \le \mathrm{Re} \le 2 \\ \frac{24}{\mathrm{Re}}\left(1 + 0.15\mathrm{Re}^{0.687}\right) & 2 \le \mathrm{Re} \le 500 \\ 0.44 & 500 \le \mathrm{Re} \le 500000 \end{cases}$$

Show that this general case reduces to the Stokes' drag equation in the limit $\mathrm{Kn} \to 0$ and $\mathrm{Re} \to 0$. (Note that saying the drag is approximately 0 in that case because the Reynolds number is small isn't good enough here. You should show after plugging in C_C and C_D that the expression you get looks exactly like the Stokes' Drag equation $F_{\text{drag}} = 3\pi\mu D_p v_t$.) You will need to use $L = D_p$ as the characteristic length of the flow.

3. In class, I asserted that the equation:

$$\tau \frac{\mathrm{d}v_z}{\mathrm{d}t} = \tau g - v_z$$

With initial condition $v_z(t=0) = 0$ can be solved via:

$$v_z(t) = \tau g \left(1 - e^{-t/\tau}\right)$$

a) Verify that $v_z(t=0) = 0$ in the above formula.

b) Verify this proposed solution satisfies the differential equation.

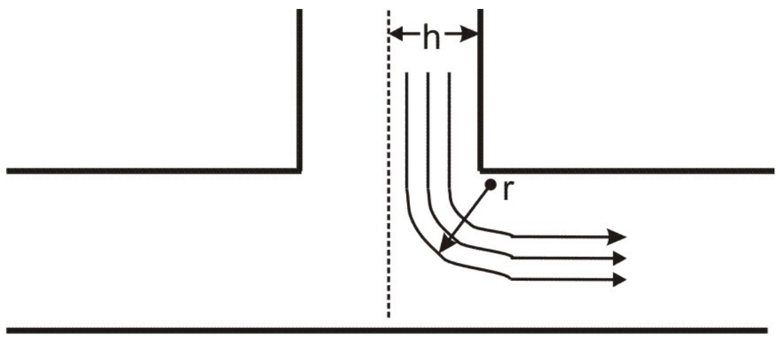
c) Find $v_z(t \to \infty)$. (Traditionally, we write $v_z(t \to \infty)$ as v_t to indicate that this is the terminal velocity of the particle.

d) How long (in terms of τ) would it take for an aerosol starting at rest to reach a fall-speed of αv_t (in other words, how long would it take to be $\alpha \times 100$ percent of the way to the terminal velocity?)

e) Given your answer to part (d), how long would it take a 100 nm diameter aerosol particle (with density 1300 kg/m^3) to reach 95% of its terminal fallspeed? (Use 1.2 as the Knudsen number for a 100 nm aerosol particle).

f) How far would the aerosol particle have fallen in part (e) in the time it takes to reach 95% of its terminal fallspeed? (Note/hint – the velocity here is not constant, so you'll actually have to do an integral to solve this successfully).

(More on next page)



4. For the simplified rectangular impactor nozzle drawn above, you may assume that the flow is uniform near the exit and that the streamlines of airflow are arcs of a circle with centers at the marked point. Some particles in the flow will move across streamlines and may be deposited on the surface.

   a) Show that particles of a given diameter, D_p, will move with constant radial velocity $v_r = (\tau U^2)/r$ where r is the radius of curvature of the streamline, τ is the relaxation time, and U is the speed of the gas. (You may assume U is constant).

   b) Show that in traveling along the arc, the particle is displaced by a total radial distance $\Delta = (\pi/2)\tau V$.

   c) Show that the fraction of particles that are deposited is $f = \frac{\pi\tau U}{2h} = \frac{\pi}{2} St$ where St is the Stokes' number.

5. To calibrate a rain-sensing device that my lab has (a 2-dimensional video disdrometer), we have a process by which we drop spherical steel ball-bearings through the device from a known height. I'm not 100% sure the exact height, but – for the sake of this problem – let's say that it is 60 cm above the sensing surface. Let's say we are dropping 10 mm spheres made out of solid steel (density of 8050 kg/m^3), initially at rest (hence "dropping") from a height of 60 cm through air (near the surface of the Earth). For these spheres, we traditionally have ignored both wind and drag. Let's make sure we haven't been making a bit mistake in my lab by ignoring these effects.

   a) Assuming $C_D = 0.44$, what is the terminal velocity (in air) of these spheres?

   b) If you neglect air resistance entirely, the downward velocity of a falling object is $v = v_\circ + gt$ and, in particular, if dropped from rest, you have $v = gt$. Use the formula derived in class to find the numerical ratio $v(t)/(gt)$ for these spheres for (i) $t = 0.1$ second, (ii) $t = 0.5$ seconds, (iii) $t = 1.0$ seconds, (iv) $t = 2.0$ seconds, (v) $t = 5.0$ seconds, and (vi) $t = 10$ seconds. (I personally used MATLAB to help me out here. Saved me a bunch of computation. You may solve this however you'd like.)

   c) Based on your answers to (a) and (b) (or other information, if necessary), explain why we don't have to factor in drag for these spheres.

6. Use your favorite computer algebra system/coding language/computational resource to draw a log-log plot of F_{drag} as a function of particle size. Have curves for Stokes' drag (dotted line) and corrected Stokes' drag (solid line) as presented in class. You may assume that the fluid is air (near the surface of the Earth) at 20°C, the fall velocity is 10 m/s, and have the x-axis (particle size) range from 0.1 nm to 1 cm. You may assume no slip-correction is necessary (e.g. $C_C = 1$). Note that this is 8 orders of magnitude difference in size, so you don't want to have a step size of 0.1 nm unless you have a week of computer time to kill. (Ask in class for hints!) In addition to the graph, please turn in your code/mathematica session/maple desktop/matlab code/excel spreadsheet/etc.