Assignment VI, PHYS 459 (Cloud and Precipitation Physics)
Fall 2021 Due Thursday, October 7th, 2021 at Beginning of Class

1. A 0.2 μm diameter particle of density 1 g/cc is being carried by an airstream at 1 atm and 298K in the $+y$ direction with a velocity of 1 m/s. The particle enters a charging device and acquires a charge of two electrons (the charge of a single electron is 1.6×10^{-19} C) and moves into a region containing a constant electric field of 10 V/m in the $+x$ direction.

   a) Calculate the relaxation time τ for the particle.

   b) Find the mean x-component of the velocity for the particle. (Hint – part (a) could be helpful!)

   c) If the region containing the electric field is 15 meters long, how far will the particle be displaced in the x-direction from its original path upon reaching the end of the electric field region?

   d) Find the answer to part (c) for a 50 nm diameter particle. (This will require you to recalculate parts (a) and (b) for a 50 nm particle). Use a particle with the same density. Note – Kn changes!

2. Assume the temperature at ground level is 300K. Also assume that we're talking about diffusion in air.

   a) Calculate the diffusion constant D for a 1 μm aerosol particle. (You may assume $C_C = 1$).

   b) How long would it take a collection of 1 μm aerosol particles to diffuse from the center to the edges of a basketball in still air at 300 K? (In other words, how long until the RMS distance traveled by each aerosol would match the radius of a basketball?)

   c) What would C_C be for a 50 nm diameter aerosol? Use $C_C = 1+\mathrm{Kn}(1.257+0.4\mathrm{e}^{-1.1/\mathrm{Kn}})$ with $\ell = 8\times10^{-7}$ m.

   d) What is the diffusion constant D for the 50 nm aerosol particle in part (c)?

   e) How long would it take a collection of 50 nm aerosol particles to diffuse from the center to the edges of a basketball in still air at 300 K? (In other words, how long until the RMS distance traveled by each aerosol would match the radius of a basketball?)

3. Consider the function $n(x,t) = n_\circ + \frac{\Delta N}{(4\pi D t)^{1/2}} \exp[-x^2/(4Dt)]$.

   a) We will verify that $n(x,t)$ above is a valid solution to the differential equation $\frac{\partial n}{\partial t} = D\frac{\partial^2 n}{\partial x^2}$. First, calculate $\frac{\partial n}{\partial t}$

   b) Now calculate $\frac{\partial n}{\partial x}$

   c) Now calculate $\frac{\partial^2 n}{\partial x^2}$

   d) Now multiply $\frac{\partial^2 n}{\partial x^2}$ by D and show you get something equivalent to $\frac{\partial n}{\partial t}$.

   e) Consider the integral $\int_{-\infty}^{\infty}[n(x,t) - n_\circ]\mathrm{d}x$. Evaluate it. (You may have to look up the "error function").

   f) What is the value of the integral $\int_{-X}^{X}[n(x,t) - n_\circ]\mathrm{d}x$? (You may leave your answer in terms of error functions and or complementary error functions.)

4. I gave you (in class) the following expression for the Diffusion Constant of an aerosol particle: $D = \frac{kTC_C}{3\pi\mu D_p}$. (Note that this expression has units of m^2/s). Another route to the diffusion equation developed from kinetic theory gives:

$$D^* = \frac{\lambda^2}{2t_{\mathrm{ave}}}$$

where λ is the mean free path between collisions for a particle, and t_{ave} is the mean time between collisions. In practice, both of these expressions are legitimate ways of writing the diffusion constant. The first method makes a bit more sense for aerosols (where there is a particle and a fluid medium that are clearly different from each other). Using D^* makes more sense if you're just talking about diffusion of a single gas into, say, a vacuum or something. This problem is about playing with the two different definitions of D simultaneously.

   a) For methane, $D = 1.78 \times 10^{-5}$ m^2/s. Assume $\lambda = 90$ nm. Find the mean time between intermolecular collisions for methane.

   b) What is the mean *instantaneous* speed for a methane molecule? (Hint....speed = distance/time).

   c) What is the root mean squared displacement from the origin for a methane molecule released at $t = 0$ at the origin and left to travel for 3 minutes?

   d) What is the actual distance that the methane molecule moved in this time? (Not the displacement, but the distance).

   e) Derive an expression for (root mean squared displacement from the origin)/(distance traveled) as a function of t. Leave your answer in terms of D, t, and v_{ave} (the average instantaneous velocity of a methane molecule).

   f) It is physically impossible for the diffusion distance to be larger than the distance traveled. This means that the expression you calculated in part (e) only can hold when $t > t_\circ$. Find the smallest possible value of $t_\circ$ for methane.

5. Under certain circumstances, the radiation inside the sun can be treated like a gas of photons with the mean free path between interactions nominally 1 mm. (We say interactions instead of collisions because we're really talking about absorption and re-emission here. Instead of "colliding" with other photons, a photon moves for nominally 1 mm before being absorbed by something. Then it is re-emitted. You may assume this absorption/re-emission process is instantaneous). (This is a rather crude model, but it works for some situations).

   a) Under this basic model, how long (on average) would it take a photon created at the center of the sun to diffuse out to the surface of the sun? (It is not, of course, the "same" photon – but let's say we're tracing the energy path instead of the photon's path. We're abusing language a bit, but the language is at least evocative). Leave your answer in years.

   b) What is the ratio:
   (root mean squared displacement from the center of the sun)/(distance traveled)
   just as the photon gets to the sun's surface?